Theory seminar
Institut für Physik, Mainz
January 24th, 2012

Black-hole lattices and inhomogeneous dust: modelling the three-dimensional universe with numerical relativity

Eloisa Bentivegna

OVERVIEW

PART I: evidence and models for Dark Energy

- Estimating cosmic distances;
- Predicting cosmic distances;
- The cosmological constant and the alternatives; averaging.

PART II: Numerical Relativity and the Einstein Toolkit

- Building 3D solutions of Einstein's equation;
- Black-hole solutions;
- The Einstein Toolkit.

PART III: black-hole lattices

- Setting up initial data;
- Evolution: apparent horizons, scaling of distances.

PART IV: inhomogeneous dust

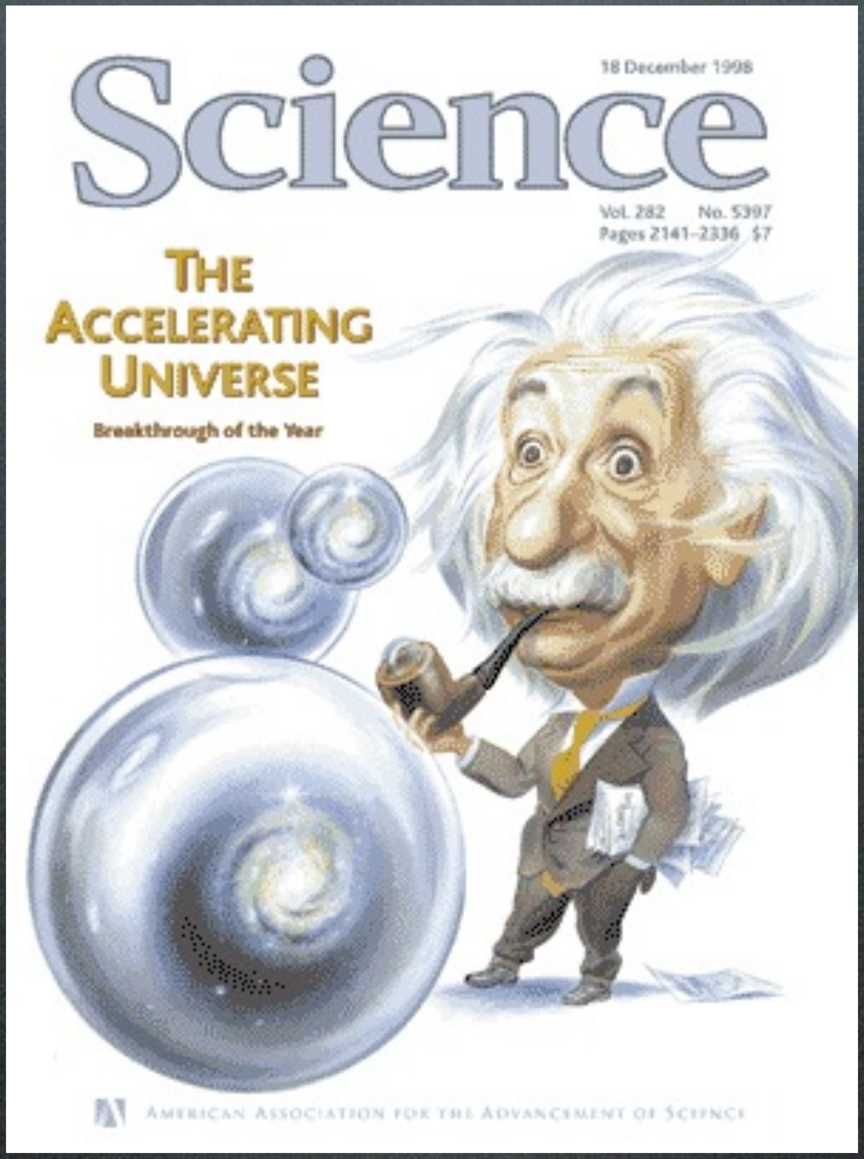
- Setting up initial data
- Evolution: growth of density contrast, volume, backreaction.

PART I

DARK ENERGY

Dark Energy

Evidence*



DARK ENERGY

EVIDENCE*

The Nobel Prize in Physics 2011
Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

Photo: Ariel Zambelich, Copyright © Nobel Media AB

Saul Perlmutter

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

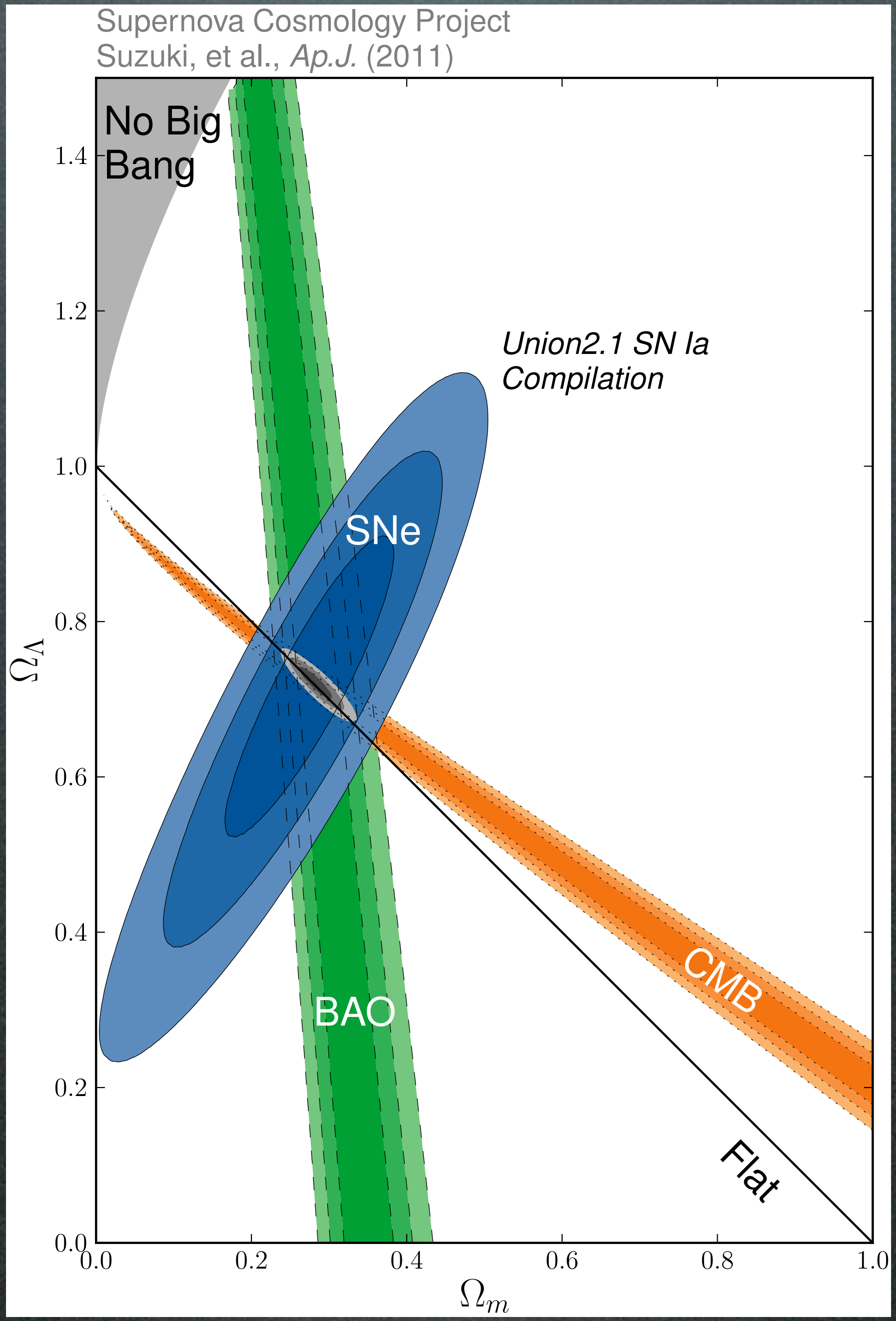
Photo: Homewood Photography

Adam G. Riess

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

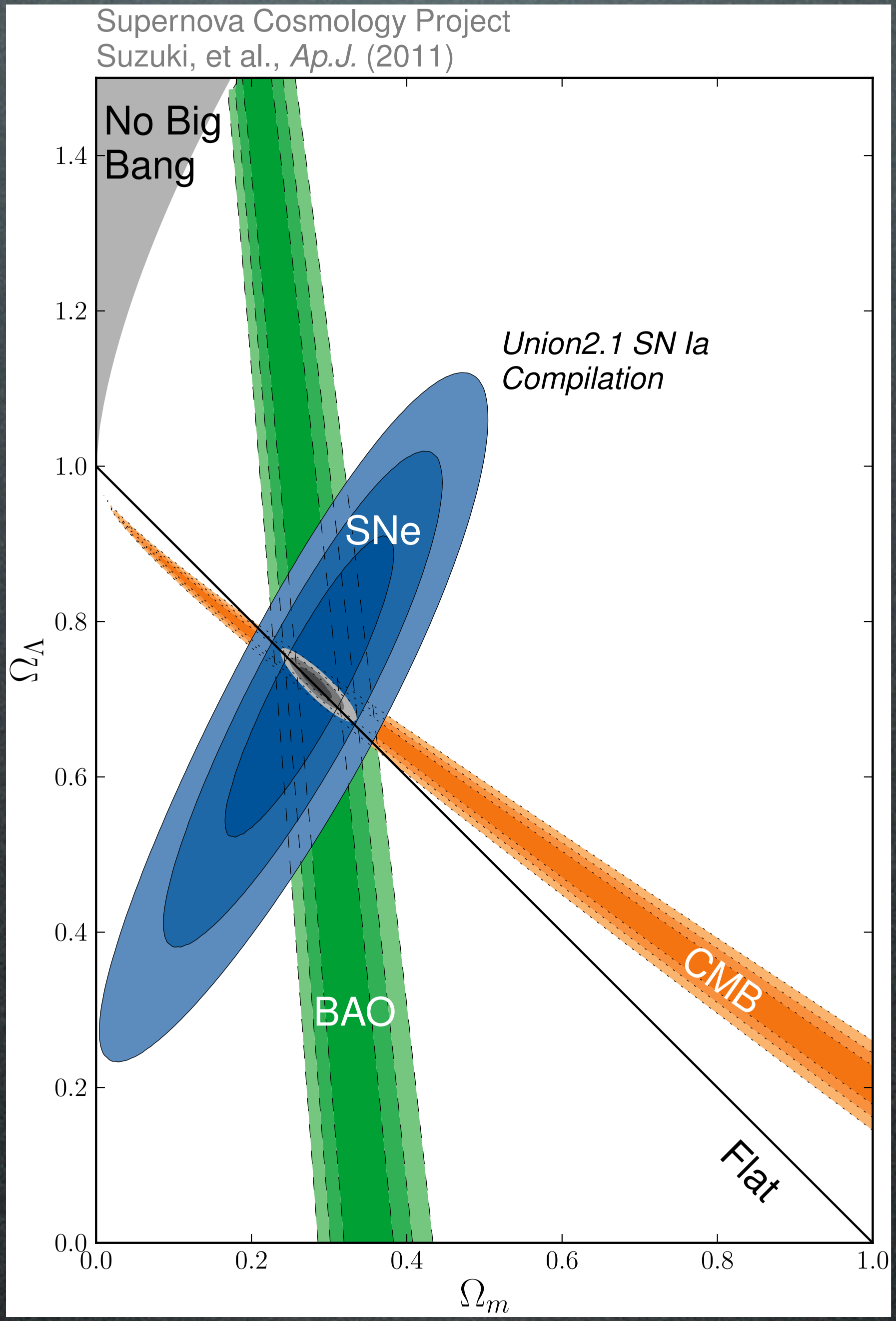
DARK ENERGY

EVIDENCE*



Dark Energy

Evidence*



*Other evidence: Nichol (2008), Sarkar (2008), both in GRG special issue on Dark Energy

DARK ENERGY

ESTIMATING COSMIC DISTANCES

Distance through the spacetime's optical properties: apparent vs. actual luminosity, apparent vs. actual size. (Need information on intrinsic properties of objects!)

Dark Energy

Estimating cosmic distances

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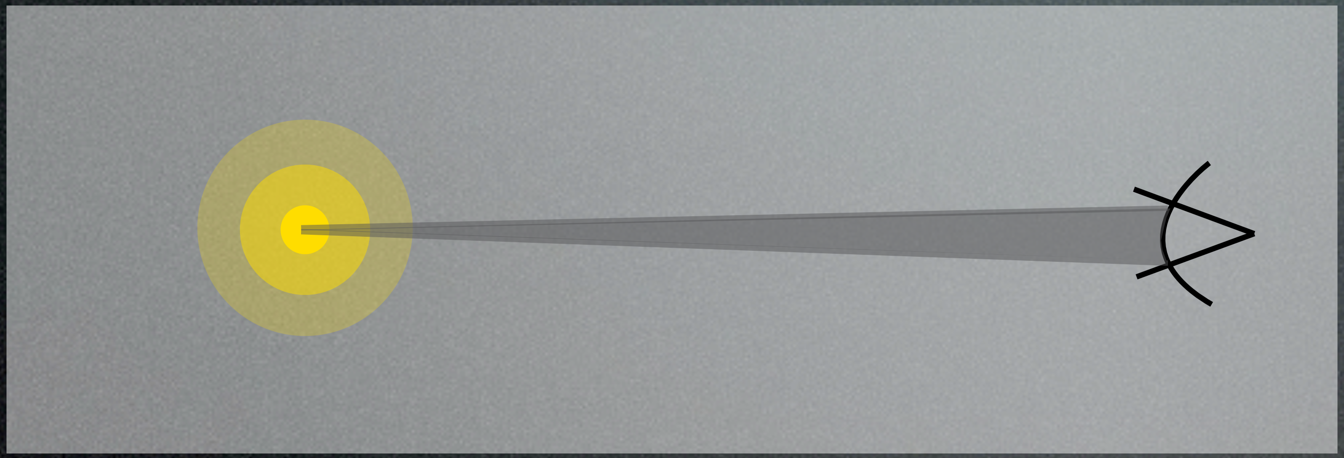

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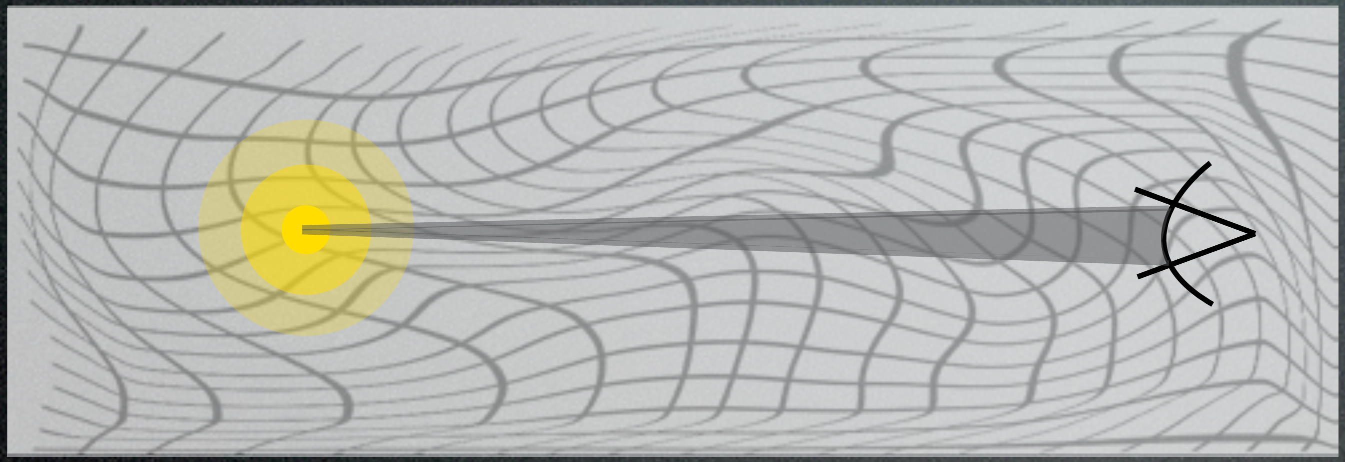

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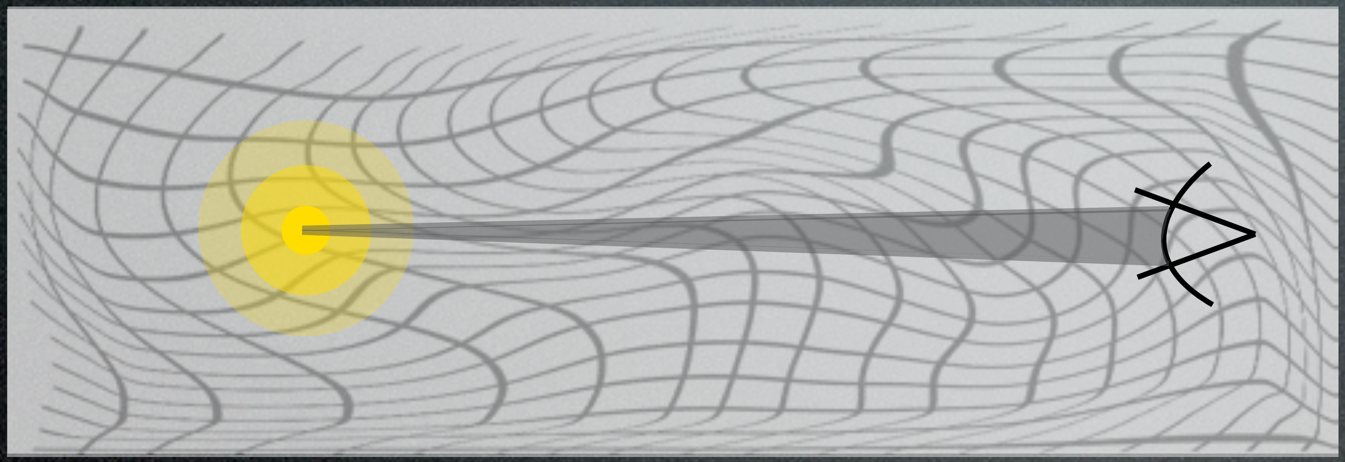

$$G_{ab} = 8\pi T_{ab} - \Lambda g_{ab}$$

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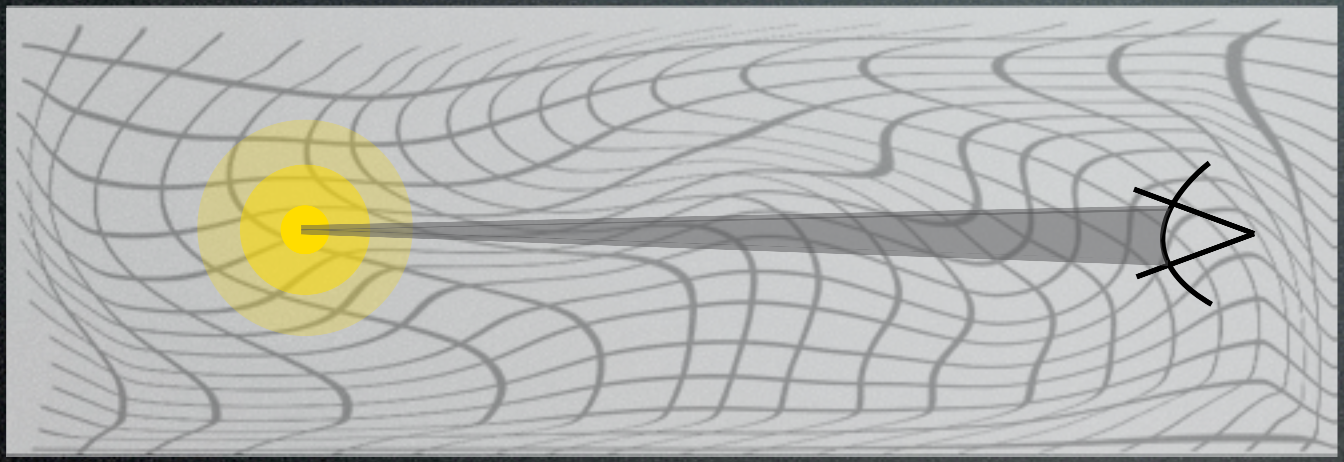

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Assumptions on T_{ab}!

Dark Energy

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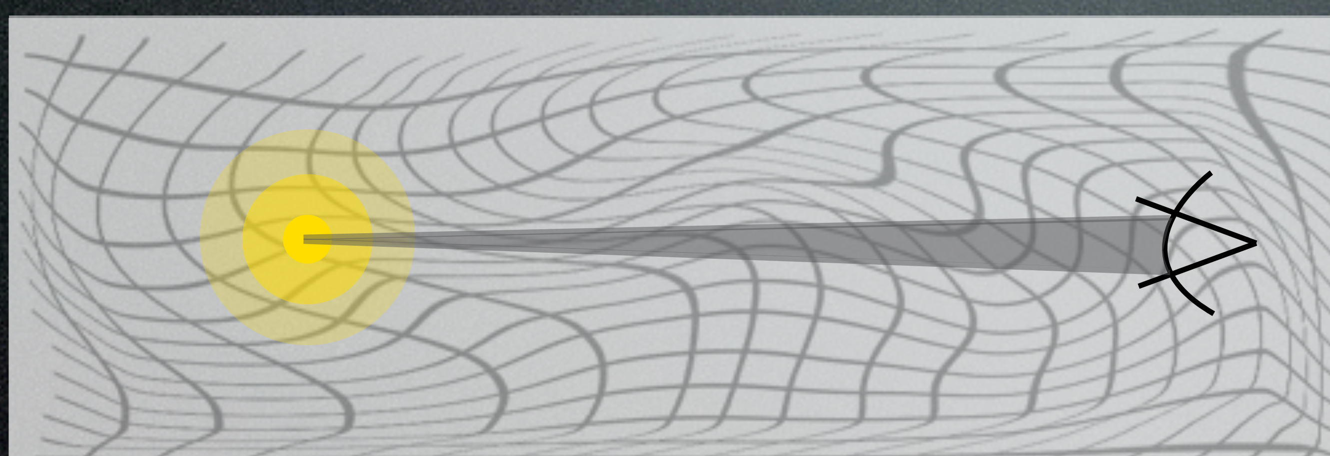
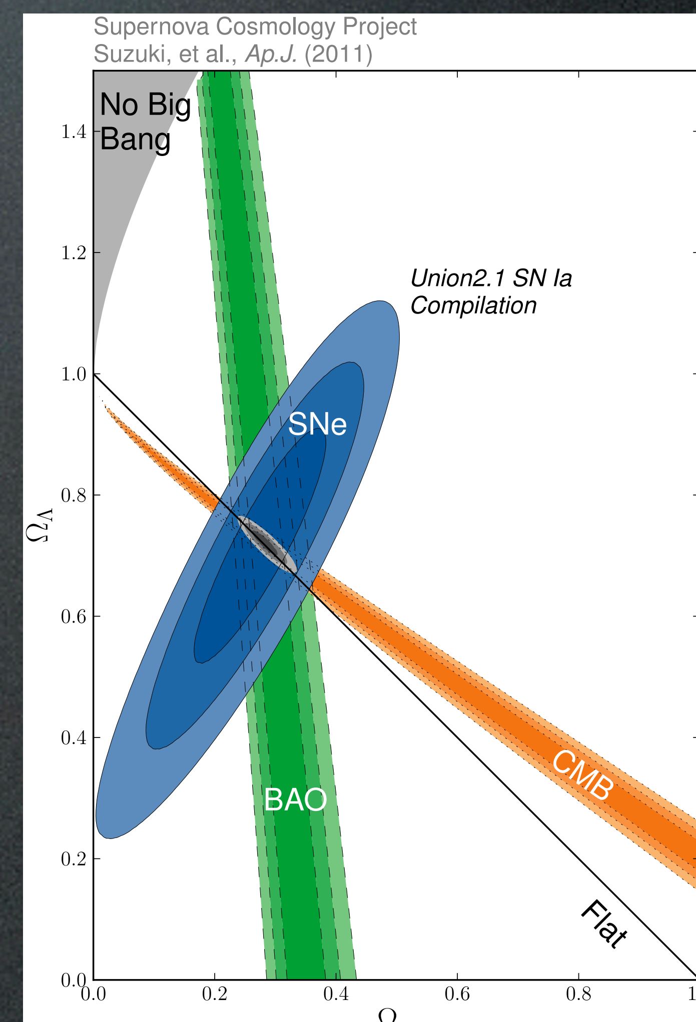




$$G_{ab} = 8\pi T_{ab} - \Lambda g_{ab}$$

Assumptions on T_{ab}!

Dark Energy

The cosmological constant problem

Whilst the cosmological-constant term is perfectly plausible, its value is suspicious:

- it's too small for vacuum energy density;
- it's strangely close to the energy density due to matter.

Solutions:

- Quintessence;
- Higher-derivative gravity and other alternative theories;
- Quantum-inspired scenarios.

Dark Energy

The fitting problem

Is this really new physics?

The fitting problem in cosmology (Ellis&Stoeger 1987): how should one map observations in a lumpy universe to exactly homogeneous and isotropic models, and what are the associated biases?

Global inhomogeneities:

- the cosmological principle is a reasonable, yet untestable ansatz.

Local inhomogeneities:

- modifications to global dynamics (the backreaction problem);
- modifications to optical properties.

Focus section on inhomogeneous cosmological models and averaging in cosmology in Classical and Quantum Gravity, August 2011.

Dark Energy

The averaging formalism

Two main approaches to modelling local inhomogeneities:

- Exact solutions
- General averaging scheme (scalars only!)

Main idea (Buchert&Ehlers 1997): define the average of a scalar field in the standard way:

$$\langle \mathcal{A}(t, X^i)\rangle_{\mathcal{D}}(t) := \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \mathcal{A}(t, X^i)\, \mathrm{d}\mu_g,$$

Introduce a congruence of observers with four-velocity field u^a, and define its expansion and shear (no rotation!):

$$\theta \equiv \nabla_i u^i \qquad \sigma_{ij} \equiv \nabla_i u_j - \frac{1}{3}\theta\gamma_{ij} \qquad \sigma^2 \equiv \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

Time evolution and averaging do not commute:

$$\boxed{\langle \mathcal{A}\rangle^{\bullet} - \langle \dot{\mathcal{A}}\rangle = \langle \mathcal{A}\theta\rangle - \langle \mathcal{A}\rangle\langle\theta\rangle}$$

Dark Energy

The averaging formalism

Volume and scale factor:

$$V_{\mathcal{D}}(t) := \int_{\mathcal{D}} J d^3 X \qquad a_{\mathcal{D}}(t) := \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_o}}\right)^{1/3}$$

The averages satisfy equations similar to those that hold in FLRW models, but with an extra contribution due to inhomogeneities:

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -4\pi G \langle \varrho \rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} + \Lambda;$$

$$3H_{\mathcal{D}}^2 + \frac{3k_{\mathcal{D}}}{a_{\mathcal{D}}^2} = 8\pi G \langle \varrho \rangle_{\mathcal{D}} - \frac{1}{2}\mathcal{W}_{\mathcal{D}} - \frac{1}{2}\mathcal{Q}_{\mathcal{D}} + \Lambda,$$

$$\mathcal{Q}_{\mathcal{D}} := \tfrac{2}{3}\left(\langle \theta^2 \rangle_{\mathcal{D}} - \langle \theta \rangle_{\mathcal{D}}^2\right) - 2\langle \sigma^2 \rangle_{\mathcal{D}},$$

$$\mathcal{W}_{\mathcal{D}} := \langle \mathcal{R} \rangle_{\mathcal{D}} - \frac{6k_{\mathcal{D}}}{a_{\mathcal{D}}^2}.$$

Dark Energy

The averaging formalism

Since the vector and tensor modes cannot be averaged, the system is not closed and there are no equations for the evolution of $\mathcal{Q}_{\mathcal{D}}$ and $\mathcal{W}_{\mathcal{D}}$ (closure relations, observations, N-body simulations). Assuming a simple power-law scaling with the scale factor, one can study the instability sectors of this system (Roy et al. 2011):

$$X_{\mathcal{D}} \equiv \mathcal{Q}_{\mathcal{D}} + \mathcal{W}_{\mathcal{D}}$$

$$a_{\mathcal{D}}^{-6}\left(\mathcal{Q}_{\mathcal{D}}\, a_{\mathcal{D}}^{6}\right)^{\cdot} + a_{\mathcal{D}}^{-2}\left(\mathcal{W}_{\mathcal{D}}\, a_{\mathcal{D}}^{2}\right)^{\cdot} = 0 .$$

$$\mathcal{Q}_{\mathcal{D}} = \mathcal{Q}_{\mathcal{D}_i} a_{\mathcal{D}}^{n} , \qquad \mathcal{W}_{\mathcal{D}} = \mathcal{W}_{\mathcal{D}_i} a_{\mathcal{D}}^{p} ,$$

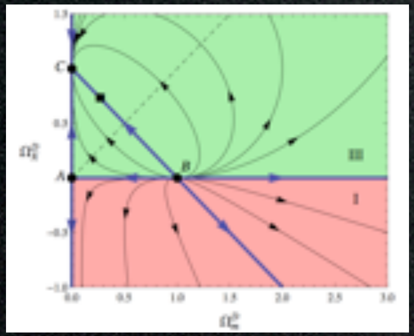
$$(n+2)\mathcal{W}_{\mathcal{D}} = -(n+6)\mathcal{Q}_{\mathcal{D}} \quad \Rightarrow \quad (n+2)X_{\mathcal{D}} = -4\mathcal{Q}_{\mathcal{D}} .$$

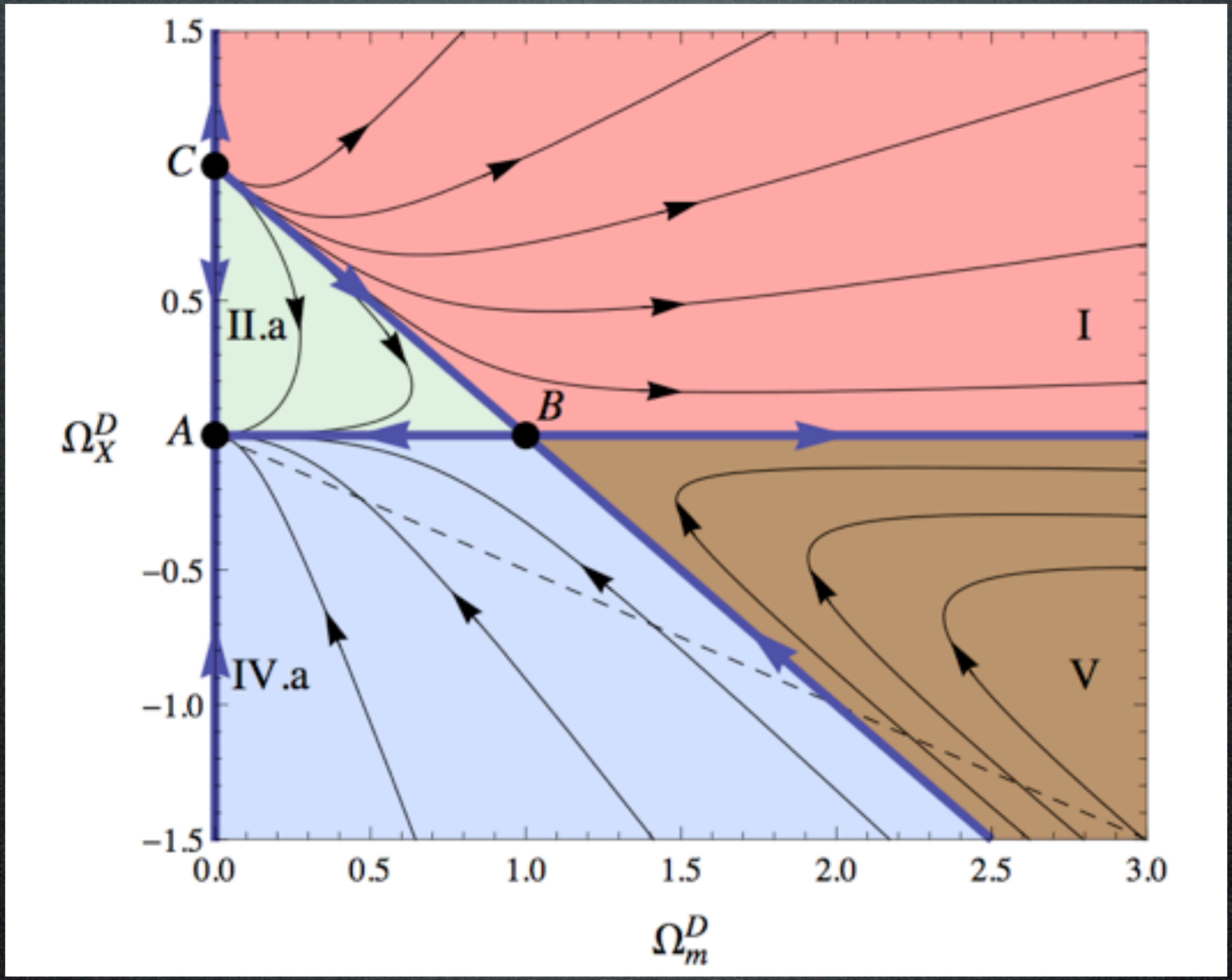
$$\begin{aligned}
&\Omega_m^{\mathcal{D}} - (n+2)\Omega_X^{\mathcal{D}} = 2q_{\mathcal{D}} , \\
&\Omega_m^{\mathcal{D}} + \Omega_k^{\mathcal{D}} + \Omega_X^{\mathcal{D}} = 1 , \\
&\Omega_m^{\mathcal{D}\,'} = \Omega_m^{\mathcal{D}}\left(\Omega_m^{\mathcal{D}} - (n+2)\,\Omega_X^{\mathcal{D}} - 1\right) , \\
&\Omega_X^{\mathcal{D}\,'} = \Omega_X^{\mathcal{D}}\left(\Omega_m^{\mathcal{D}} - (n+2)\,\Omega_X^{\mathcal{D}} + n + 2\right)
\end{aligned}$$

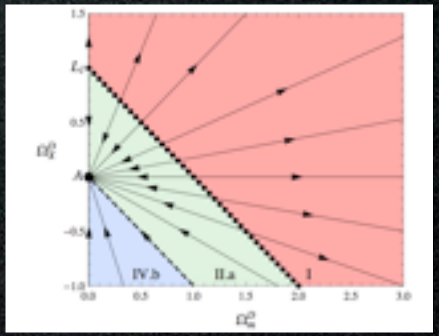
$$\Omega_m^{\mathcal{D}} := \frac{8\pi G}{3H_{\mathcal{D}}^2}\langle\varrho\rangle_{\mathcal{D}} , \qquad \Omega_k^{\mathcal{D}} := -\frac{k_{\mathcal{D}_i}}{a_{\mathcal{D}}^2\, H_{\mathcal{D}}^2} , \qquad \Omega_X^{\mathcal{D}} := -\frac{X_{\mathcal{D}}}{6H_{\mathcal{D}}^2} ,$$

Dark Energy

The averaging formalism







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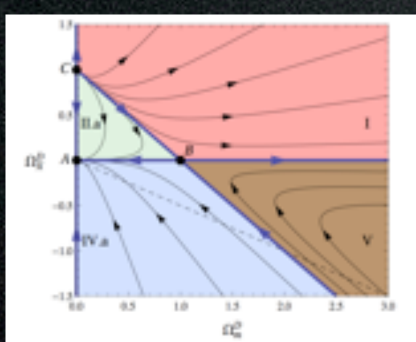

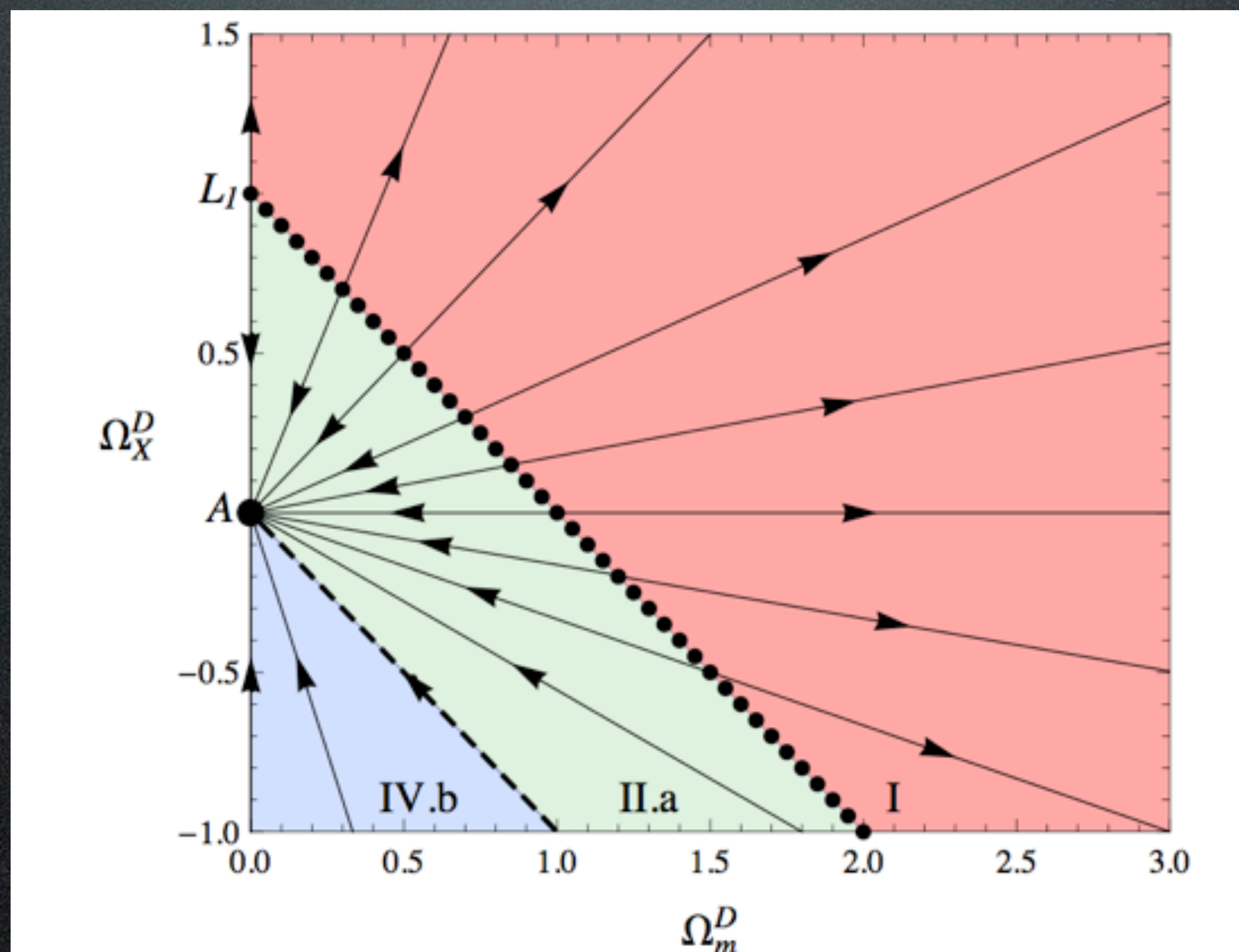

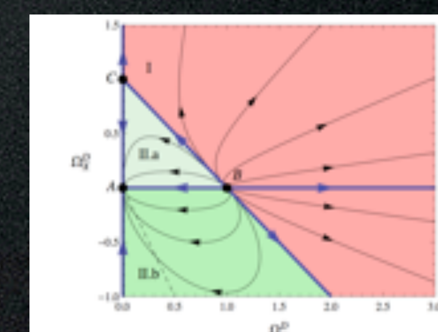

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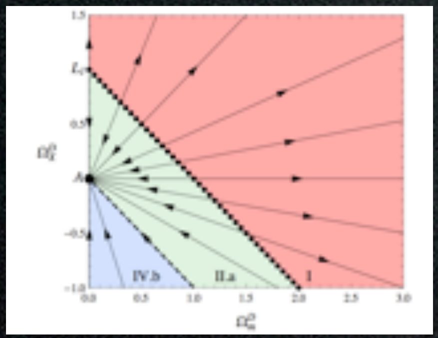

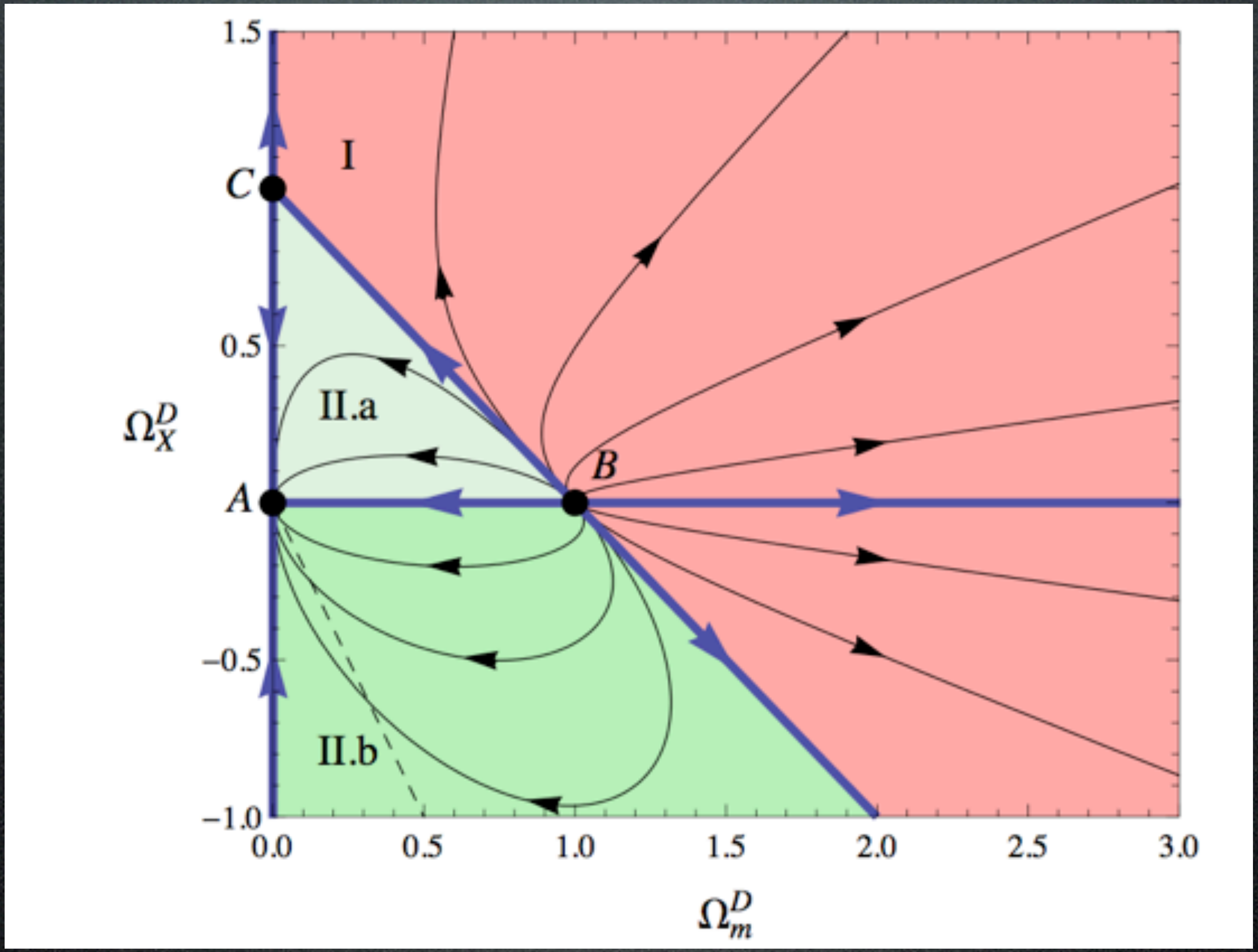

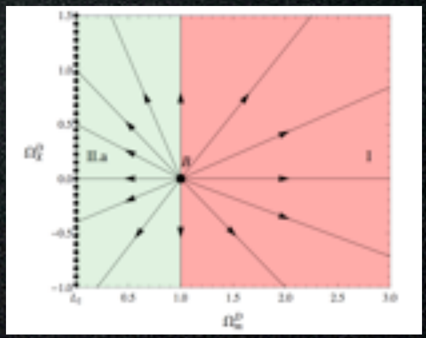

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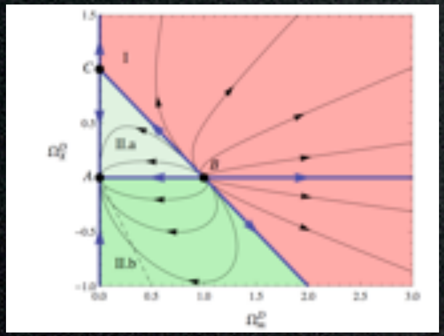

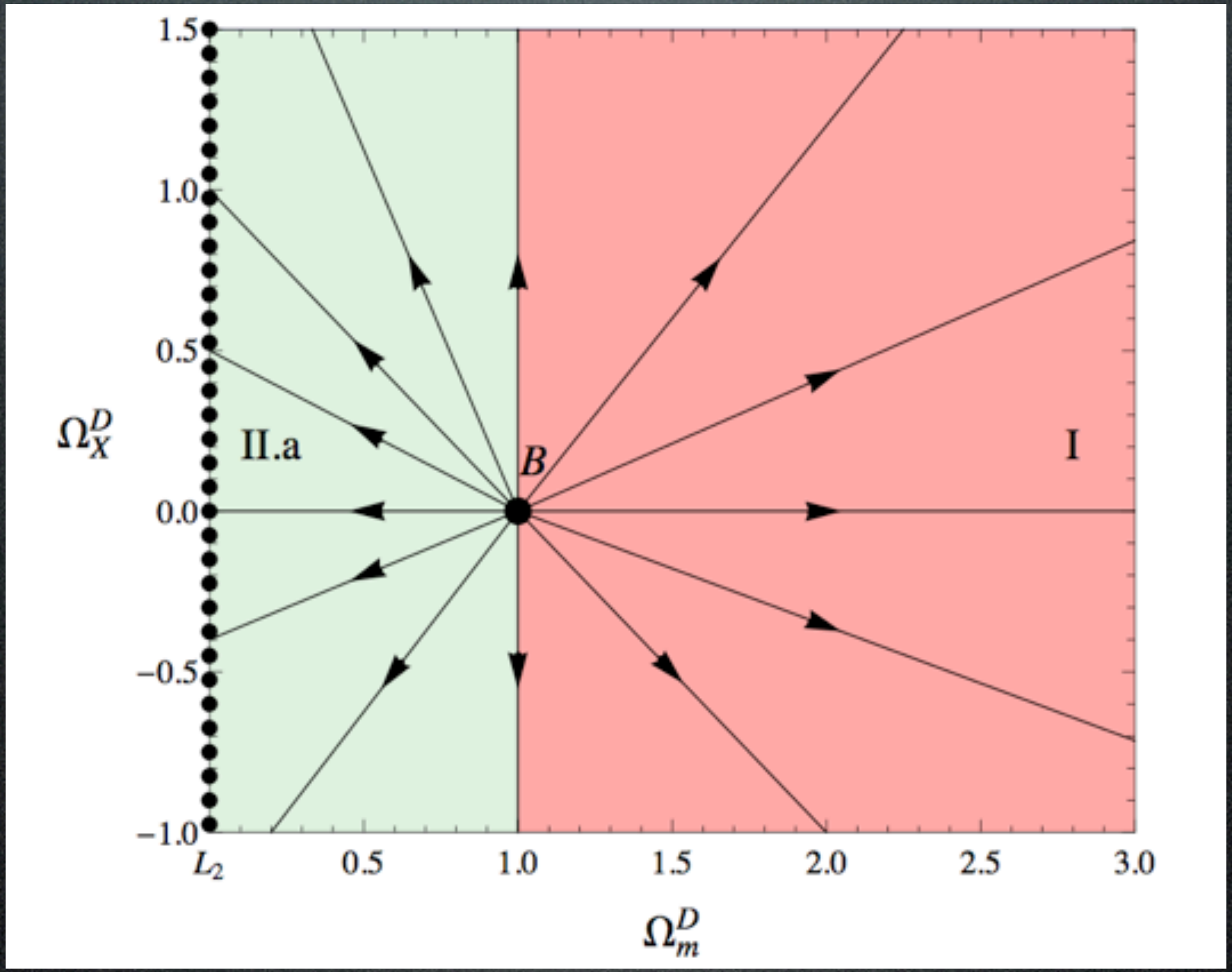

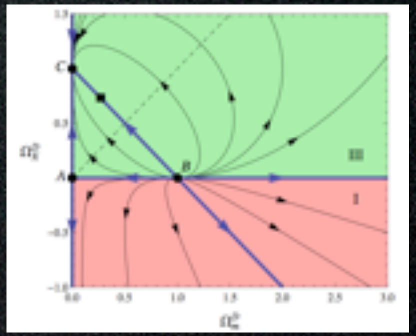

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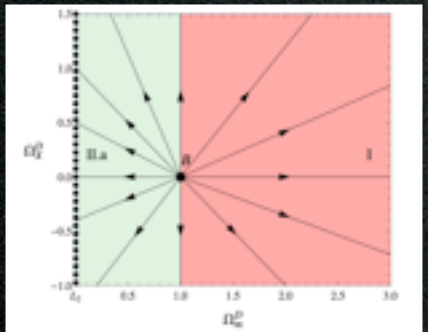

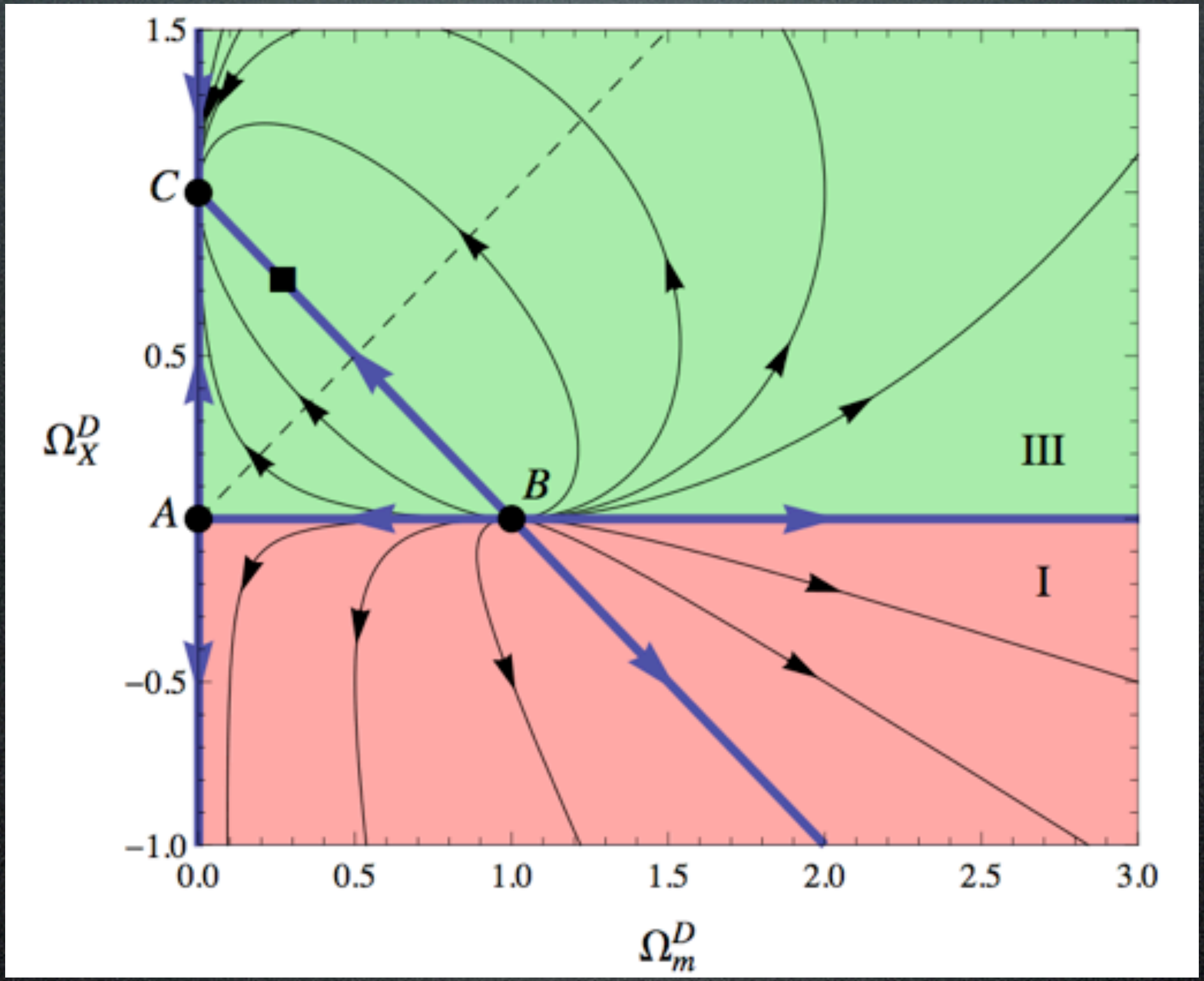

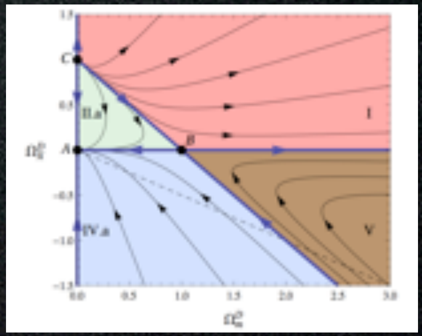

DARK ENERGY

COSMOTOOLKIT

Bridging the gap: construct 3D, exact solutions using the formalism and tools of numerical relativity:

- Expressing Einstein's equation as a system of PDEs suitable for numerical integration
- Constructing initial data of cosmological relevance
- Carrying out the evolution explicitly

Currently tackling two classes of solutions:

- Regular lattices of black holes;
- Perfect fluid with inhomogeneities

Developing:

- Elliptic solver;
- Mesh refinement;
- Ray tracing and optical properties;
- ...

Marie Curie PIRG05-GA-2009-249290 (2010-2013)

PART II

NUMERICAL RELATIVITY AND THE EINSTEIN TOOLKIT

Numerical Relativity

The 3+1 decomposition

In order to formulate Einstein's equation as an initial-boundary value problem, one needs to choose a time coordinate and project the equations accordingly; reducing the system to first order form, one is left with twelve evolution four constraints equations:

NUMERICAL RELATIVITY

THE 3+1 DECOMPOSITION

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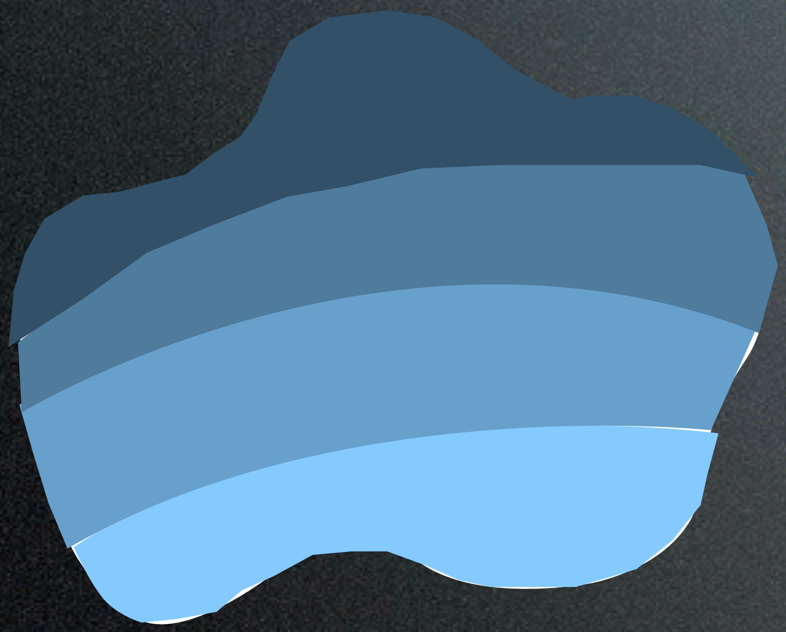

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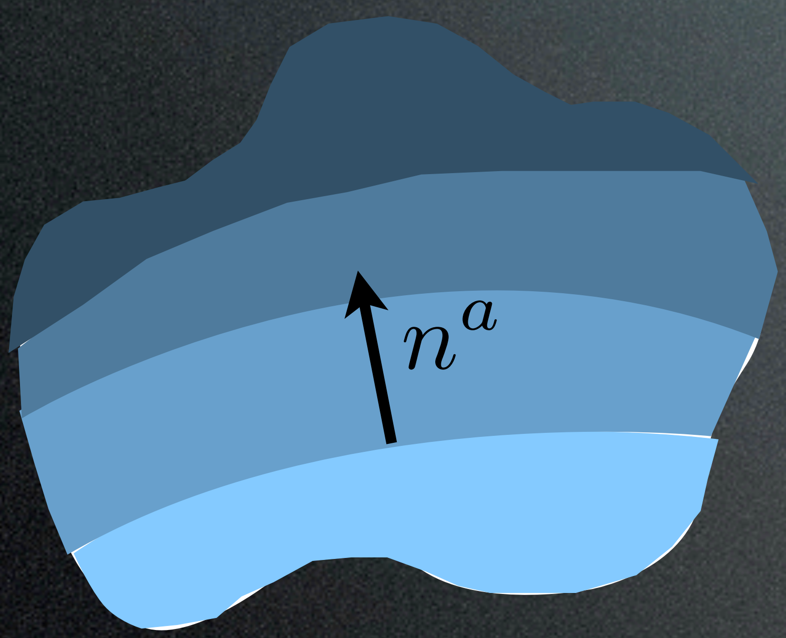

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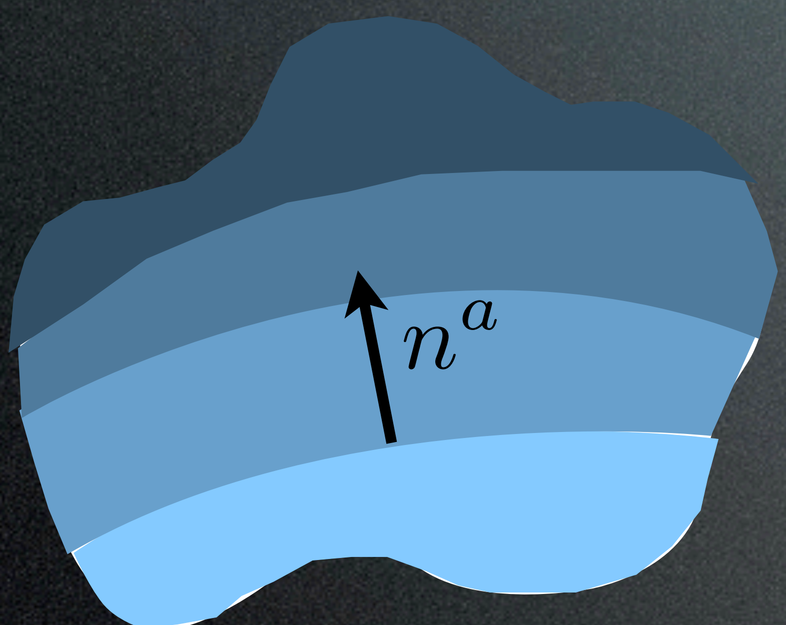

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

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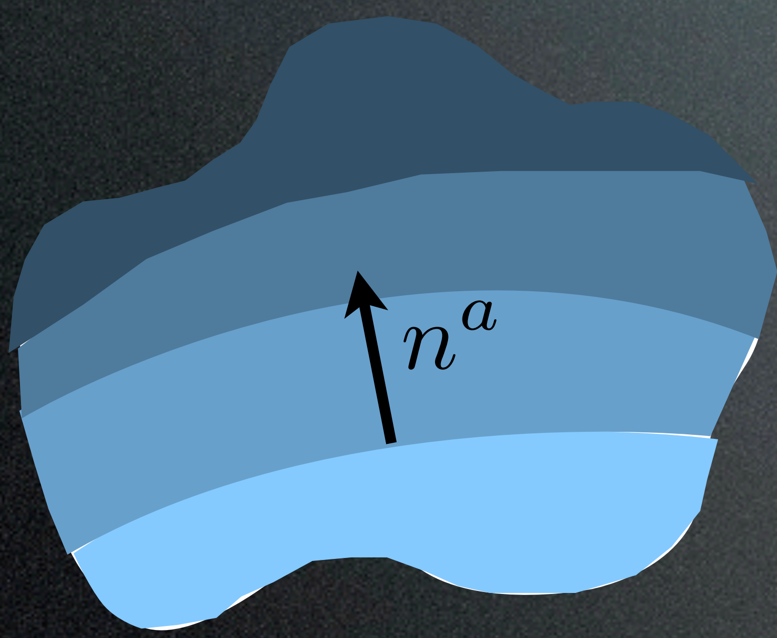

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\gamma_{ij} = g_{ij} + n_i n_j$$

$$K_{ij} \equiv -\mathcal{L}_n \gamma_{ij}$$

NUMERICAL RELATIVITY

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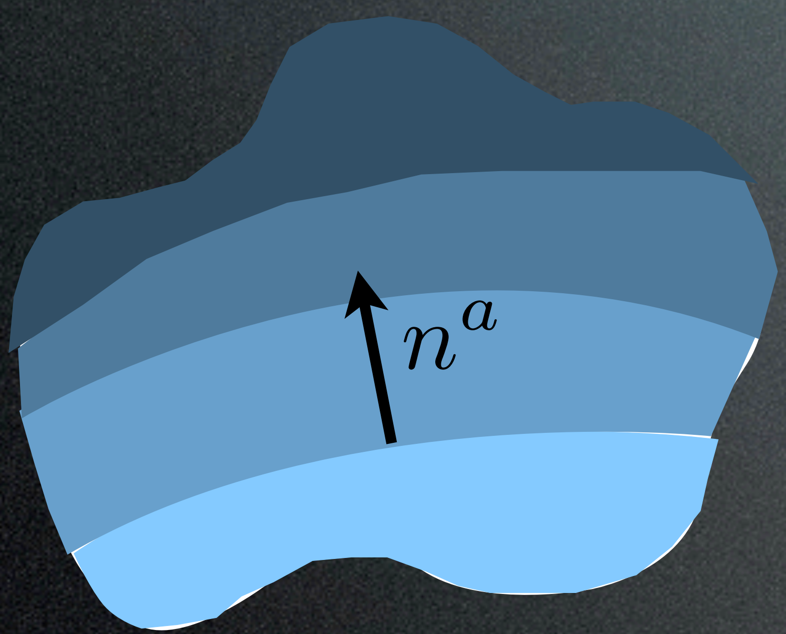

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$$K_{ij} \equiv -\mathcal{L}_n \gamma_{ij}$$

$$\begin{aligned} R + K^2 - K_{ij}K^{ij} &= 0 \\ D_j K^j_i - D_i K &= 0 \end{aligned}$$

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\partial_t K_{ij} = -D_i D_j \alpha + \alpha(R_{ij} - 2K_{ik}K^k_j + KK_{ij}) + \beta^k D_k K_{ij} + K_{ik} D_j \beta^k + K_{kj} D_i \beta^k$$

NUMERICAL RELATIVITY

THE EINSTEIN TOOLKIT

The Einstein Toolkit:

- Open-source toolkit;
- One code-generating framework;
- Over one hundred components (evolution of the gravitational field and fluids, analysis of spacetimes, I/O);
- AMR capabilities;
- Leveraging HPC systems worldwide;
- Tutorials and demos for new users — try it out!

NUMERICAL RELATIVITY

BLACK-HOLE SOLUTIONS

Black holes represent the most basic class of non-trivial, vacuum, exact solutions of Einstein's equation.

The most distinctive feature of these spacetimes is the presence of regions causally disconnected with distant observers. As a norm, "trapped" surfaces will be part of these spacetimes too, and provide a useful analysis tool because:

- They are local;
- They obey a few laws of black-hole mechanics.

In particular, one can associate meaningful definitions of mass and angular momentum to these local structures.

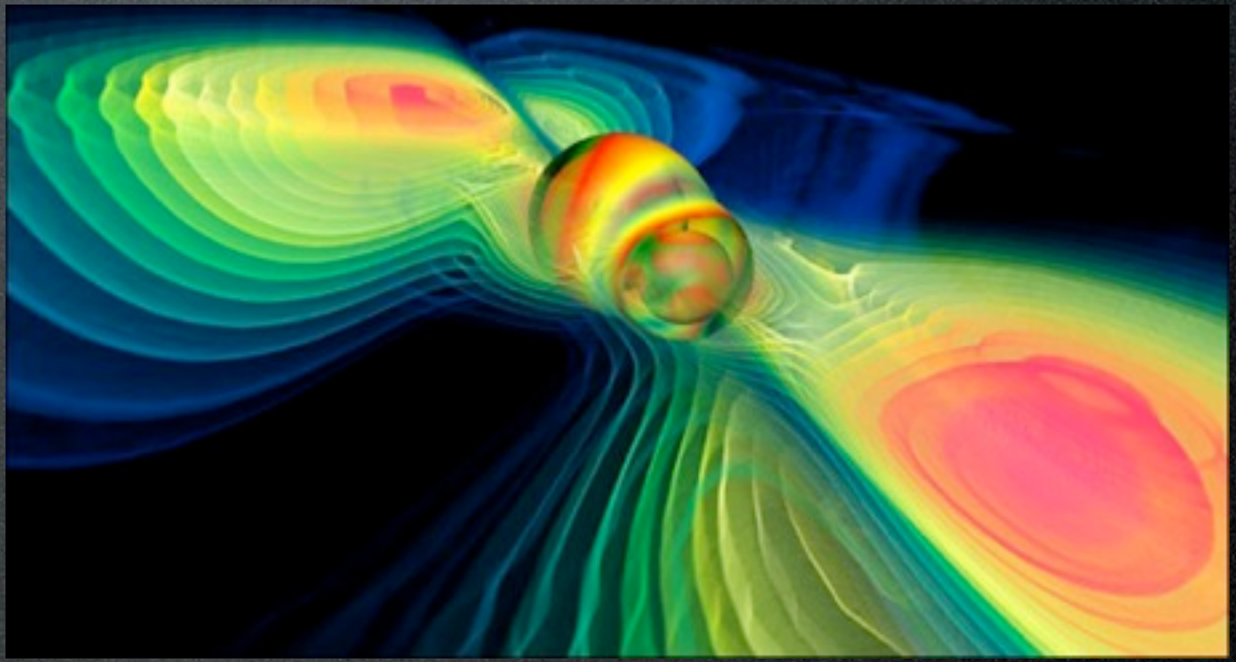
Numerical Relativity

State of the art

Black holes also play an important observational role, since they drive galactic activity and evolution, are thought to be at the core of a class of Gamma Ray Bursts, and are powerful gravitational-wave emitters when excited.

Thanks to numerical relativity, a number of these scenarios have been under scrutiny in the last ten years, with many more being actively pursued now:

- Black-hole binaries
- Neutron-star binaries
- Mixed binaries
- Gravitational collapse and supernovae
- Black holes surrounded by accretion disks



MPI for Gravitational Physics/W.Benger-ZIB

Part III

Black-hole lattices

With Mikołaj Korzyński

Black-hole lattices

Constructing a black-hole lattice

Models studied by Lindquist&Wheeler (1957) and Clifton&Ferreira (2009-2011) via embeddings of the Schwarzschild solution in uniform-curvature spaces.

Exact initial data: in order to solve the initial-boundary value problem in General Relativity, one must specify a valid initial dataset, namely one that satisfies the Einstein constraints:

$$\begin{aligned} R + K^2 - K_{ij}K^{ij} &= 0 \\ D_i K^i_j - D_j K &= 0 \end{aligned}$$

$$\begin{aligned} \gamma_{ij} &= \psi^4 \, \tilde{\gamma}_{ij} \\ K_{ij} &= \frac{K}{3}\,\gamma_{ij} + A_{ij} \end{aligned}$$

$$\begin{aligned} \tilde{\Delta}\psi - \tilde{R}\psi - \frac{K^2}{12}\,\psi^5 + \frac{1}{8}\tilde{A}_{ij}\tilde{A}^{ij}\psi^{-7} &= 0 \\ \tilde{D}_i\tilde{A}^{ij} - \frac{2}{3}\psi^6\tilde{\gamma}^{ij}\tilde{D}_i K &= 0 \end{aligned}$$

BLACK-HOLE LATTICES

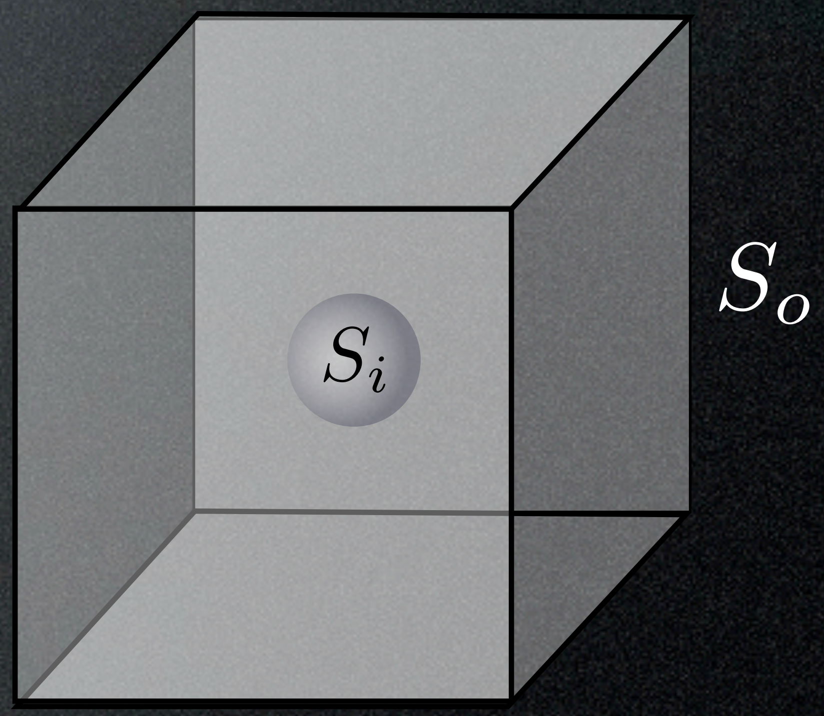
CONSTRUCTING A BLACK-HOLE LATTICE

Further constraint: if we assume the following form for the conformal factor:

$$\psi = 1 + \frac{M}{r}$$

then the extrinsic curvature and the scalar curvature cannot both be zero, otherwise:

$$0 = \int_V \tilde{\Delta}\psi = -\int_{S_i} \tilde{\Delta}\psi + \int_{S_o} \tilde{\Delta}\psi = -M$$



Black-hole lattices

Constructing a black-hole lattice

Two options:

- Keep a zero extrinsic curvature, but choose a conformal metric that is not flat:

$$\tilde{\Delta}\,\psi - \frac{\tilde{R}}{8}\,\psi = 0$$

  Note: the hamiltonian constraint is linear! One can use the superposition principle to construct multi-black-hole solutions.

- Keep a flat conformal metric, but use a non-zero extrinsic curvature

$$\Delta\psi - \frac{K^2}{12}\psi^5 = 0$$

  Requires numerical integration.

BLACK-HOLE LATTICES

CONFORMALLY-S3 BLACK-HOLE LATTICES

Hamiltonian constraint:

$$\tilde{\Delta}\,\psi - \frac{\tilde{R}}{8}\,\psi = 0$$

Spatial metric given by:

$$\mathrm{d}s^2 = \mathrm{d}\lambda^2 + \sin^2\lambda\,\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

Single-black-hole solution:

$$\phi(\lambda) = \frac{A}{\sin\lambda/2}$$

Black-hole lattices

Conformally-S3 black-hole lattices

Multiple black holes can be obtained by superimposing this fundamental solution. It is convenient to embed this three-sphere in R4, and to express the solution in this coordinate space:

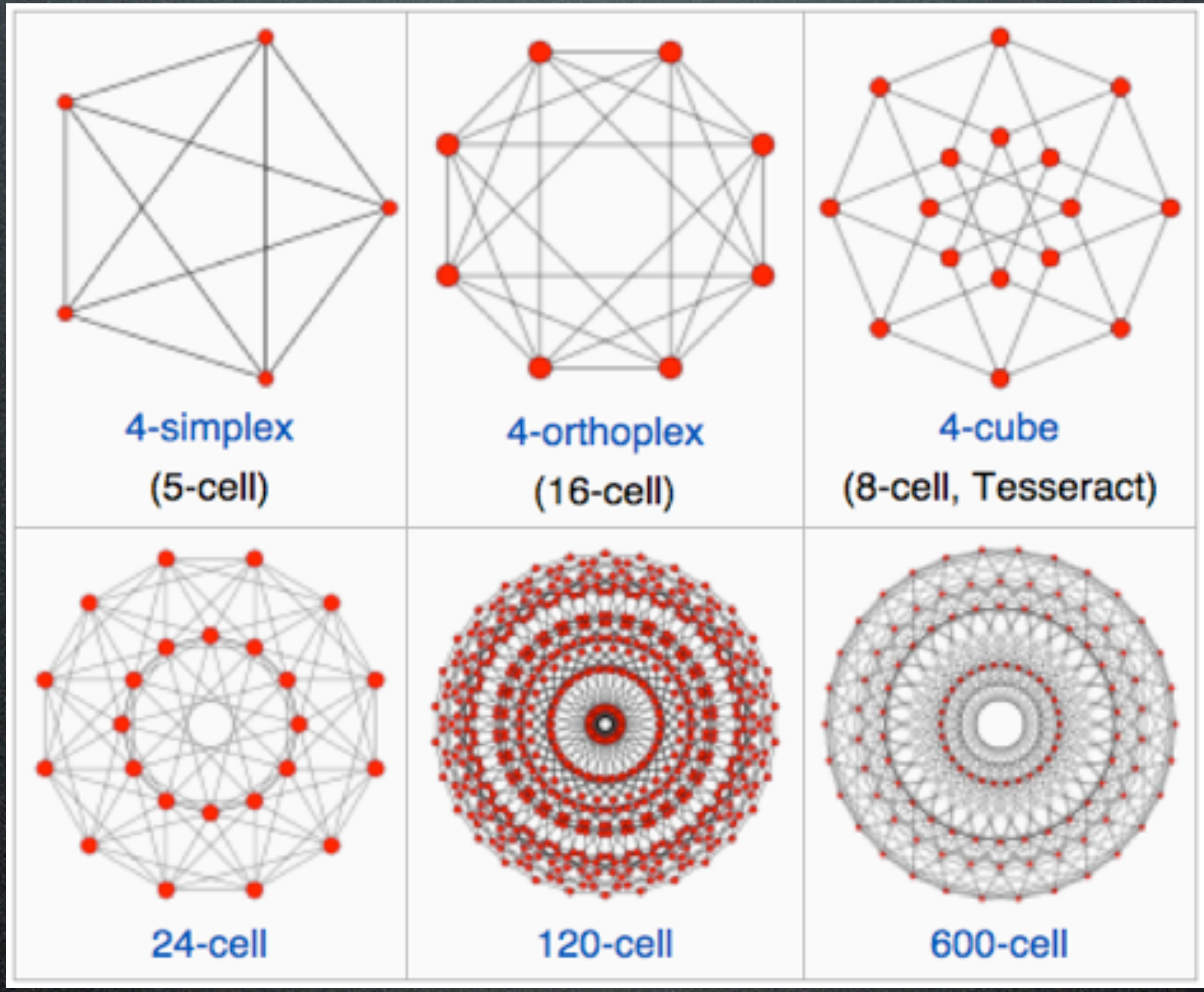
$$\phi(\bar{X}) = \sum_{i=1}^{N} \frac{A_i}{\sin \lambda_i / 2} = \sum_i A_i \sqrt{\frac{2}{1 - \bar{X} \cdot \bar{N}_i}}$$

The parameters A_i and the black-hole centers are arbitrary, but if one is interested in regular lattices these have to be chosen carefully. In particular, the parameters A_i have to be the same, and the centers have to be equidistant from each other.

Black-hole lattices

The regular tessellations of S3

On a three-sphere, there is only a finite number of "regular" arrangements of points, corresponding to the regular tessellations of S^3. N can be equal to 5, 8, 16, 24, 120, 600.

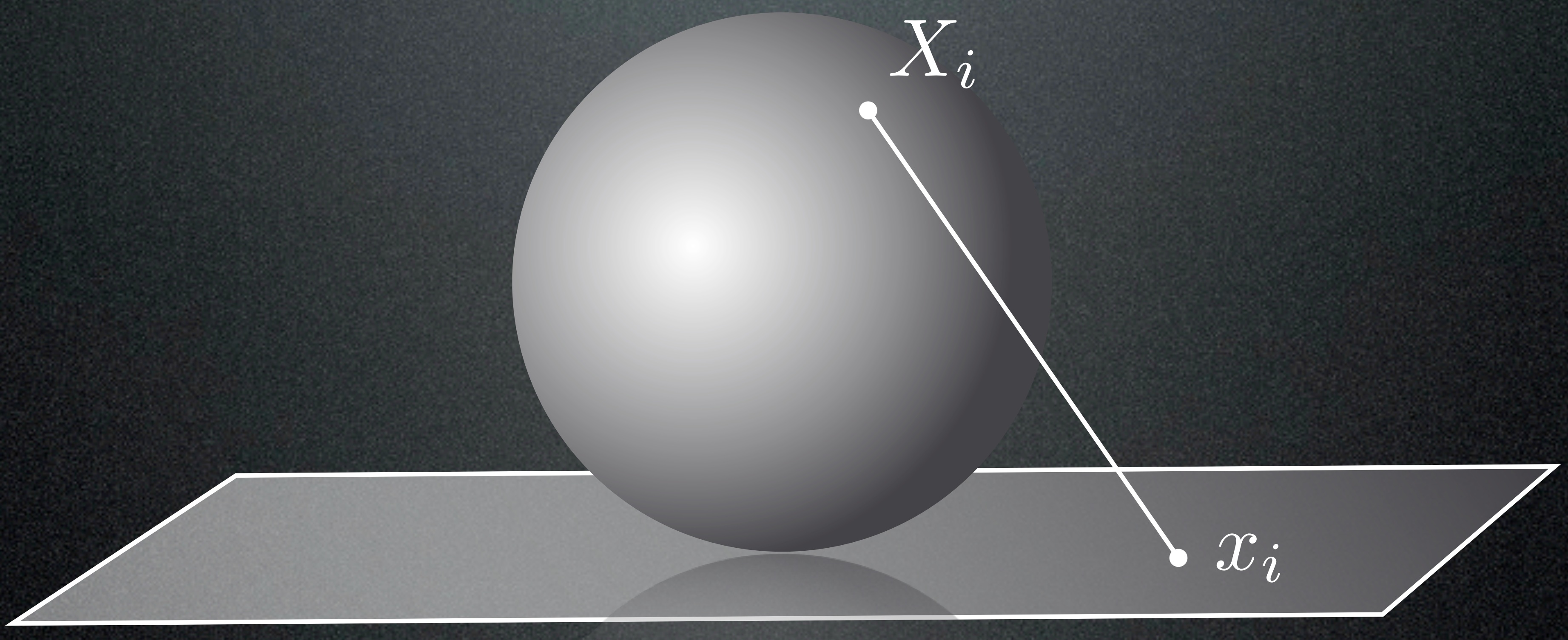


Black-hole lattices

A stereographic projection from S^3 onto R^3

Coordinate transformation to simplify numerical treatment:

$$(X^1, X^2, X^3, X^4) \to (x^1, x^2, x^3) : x^i = \frac{2X^i}{1 - X^4}$$

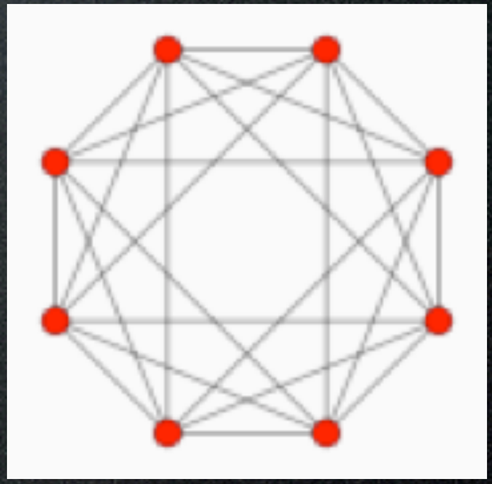


Black-hole lattices

The 8-black-hole universe

We have studied the 8-black-hole case and its full-GR evolution:

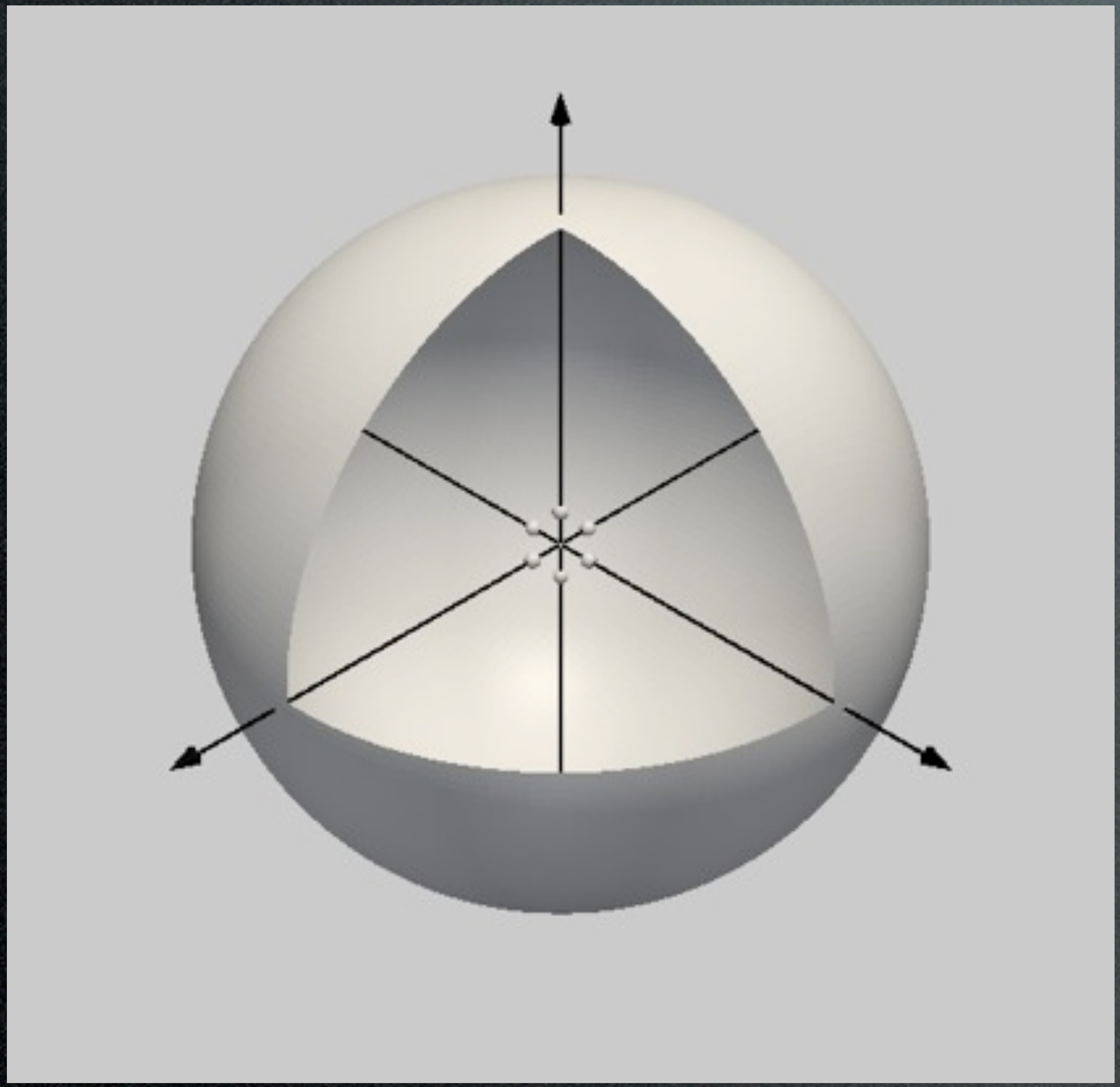
$$
\begin{aligned}
\bar{N}_1 &= (1,0,0,0)\,, \\
\bar{N}_2 &= (-1,0,0,0)\,, \\
\bar{N}_3 &= (0,1,0,0), \\
\bar{N}_4 &= (0,-1,0,0), \\
\bar{N}_5 &= (0,0,1,0)\,, \\
\bar{N}_6 &= (0,0,-1,0)\,, \\
\bar{N}_7 &= (0,0,0,1), \\
\bar{N}_8 &= (0,0,0,-1).
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
\vec{\mathcal{N}}_2 &= (0,0,0)\,, \\
\vec{\mathcal{N}}_3 &= (2,0,0), \\
\vec{\mathcal{N}}_4 &= (-2,0,0), \\
\vec{\mathcal{N}}_5 &= (0,2,0)\,, \\
\vec{\mathcal{N}}_6 &= (0,-2,0)\,, \\
\vec{\mathcal{N}}_7 &= (0,0,2), \\
\vec{\mathcal{N}}_8 &= (0,0,-2).
\end{aligned}
$$

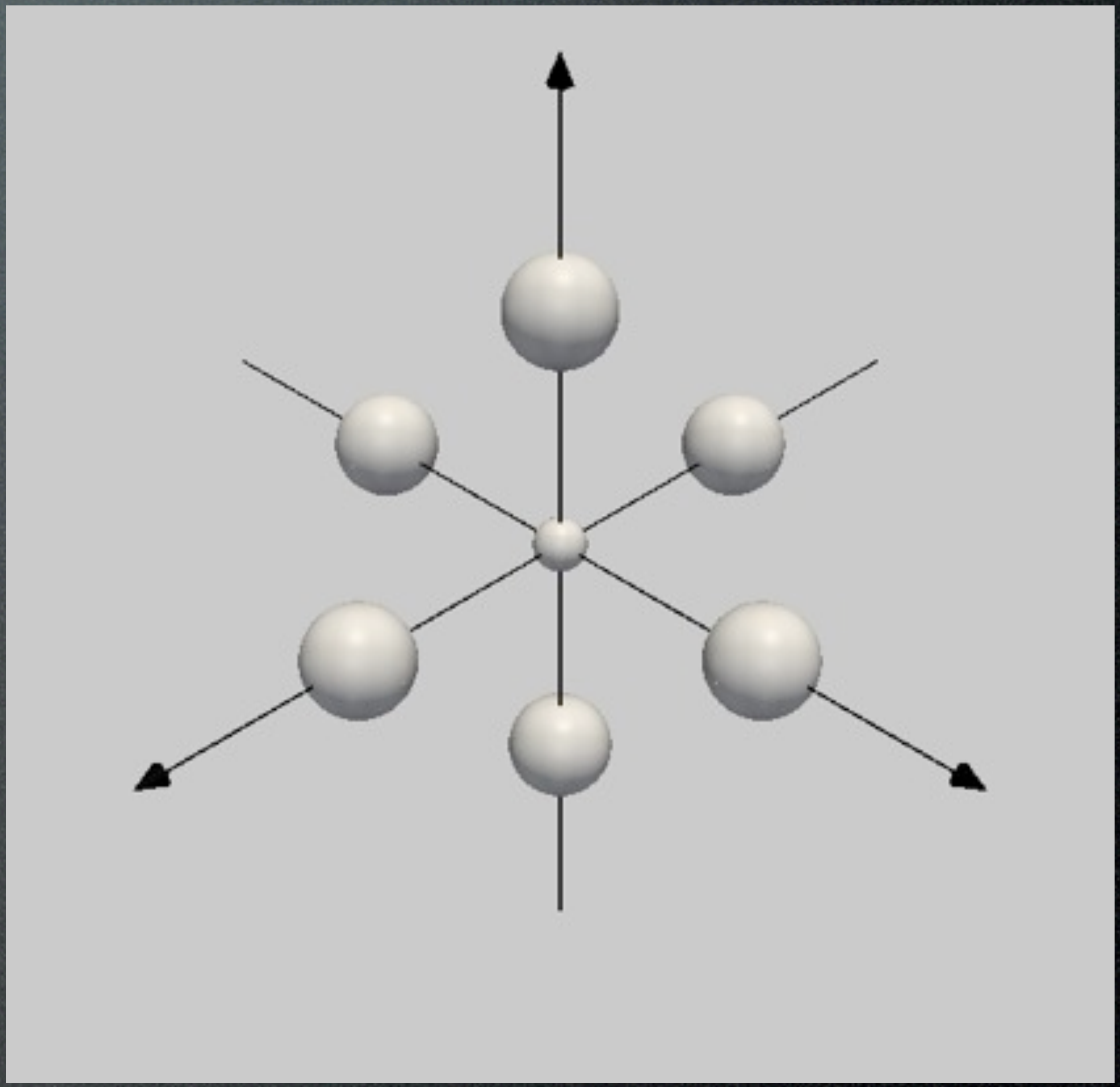


Black-hole lattices

The 8-black-hole universe

Initial horizons:





Black-hole lattices

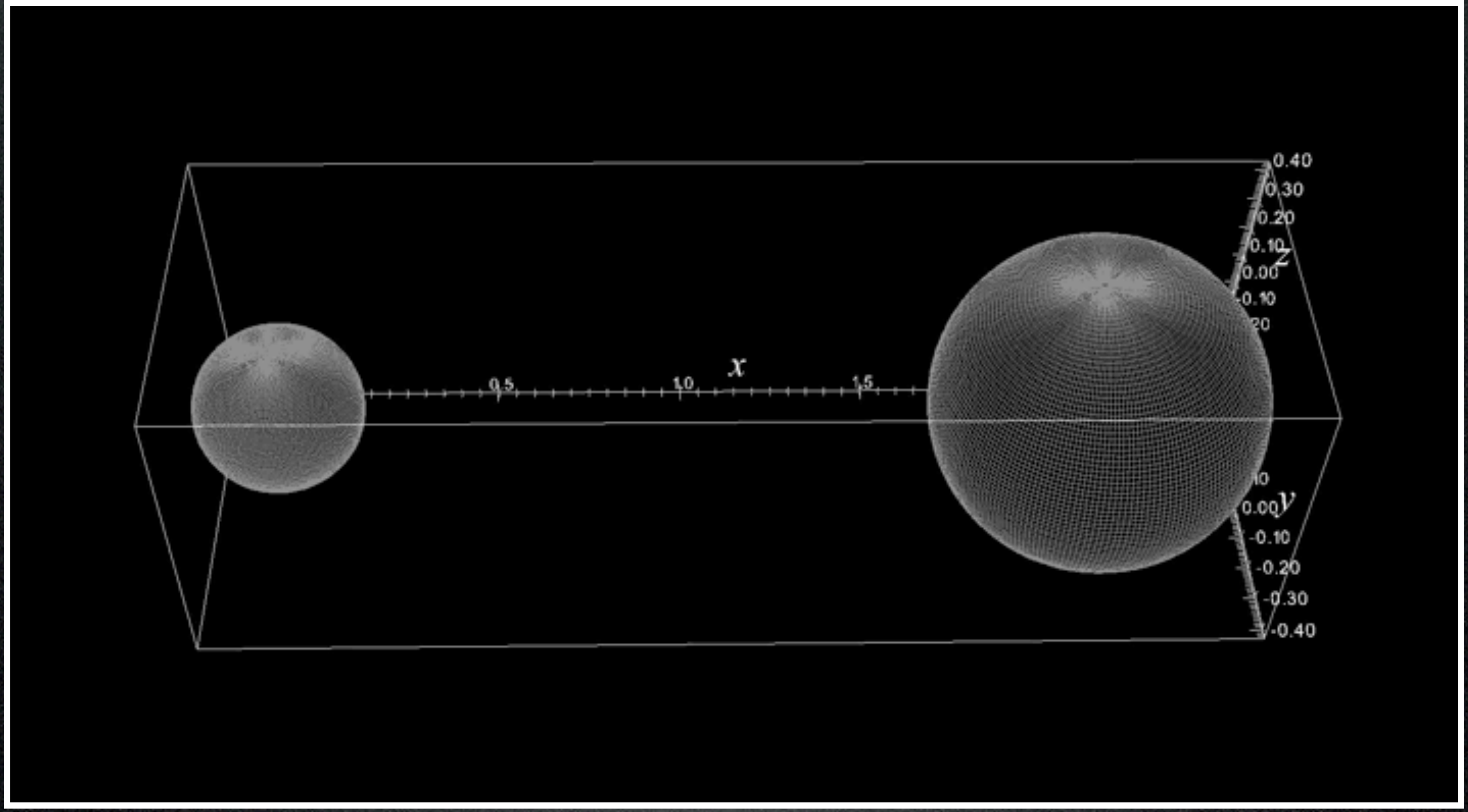
The 8-black-hole universe

Horizon evolution:

BLACK-HOLE LATTICES

THE 8-BLACK-HOLE UNIVERSE

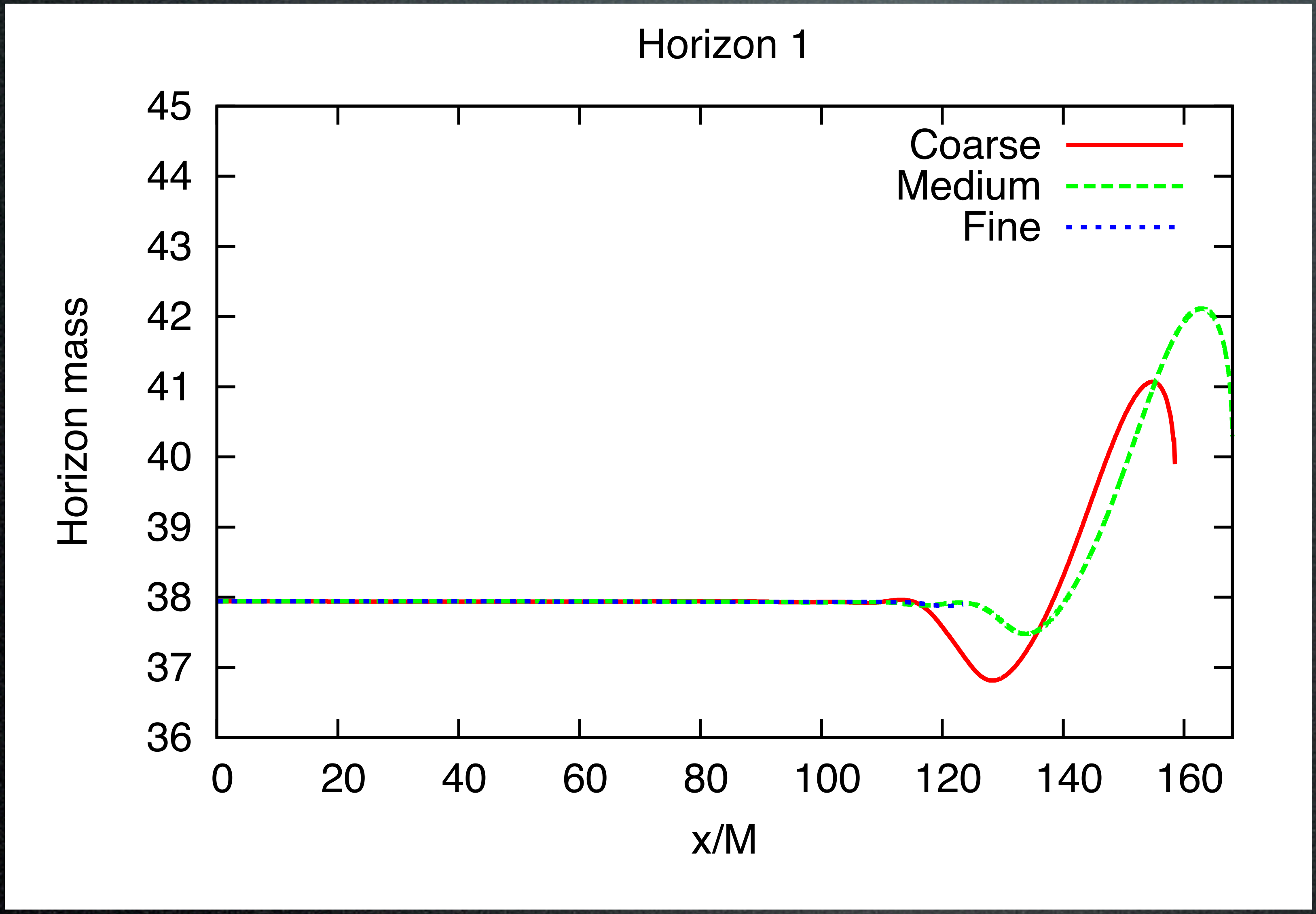
Horizon evolution:



Black-hole lattices

The 8-black-hole universe

Horizon properties:

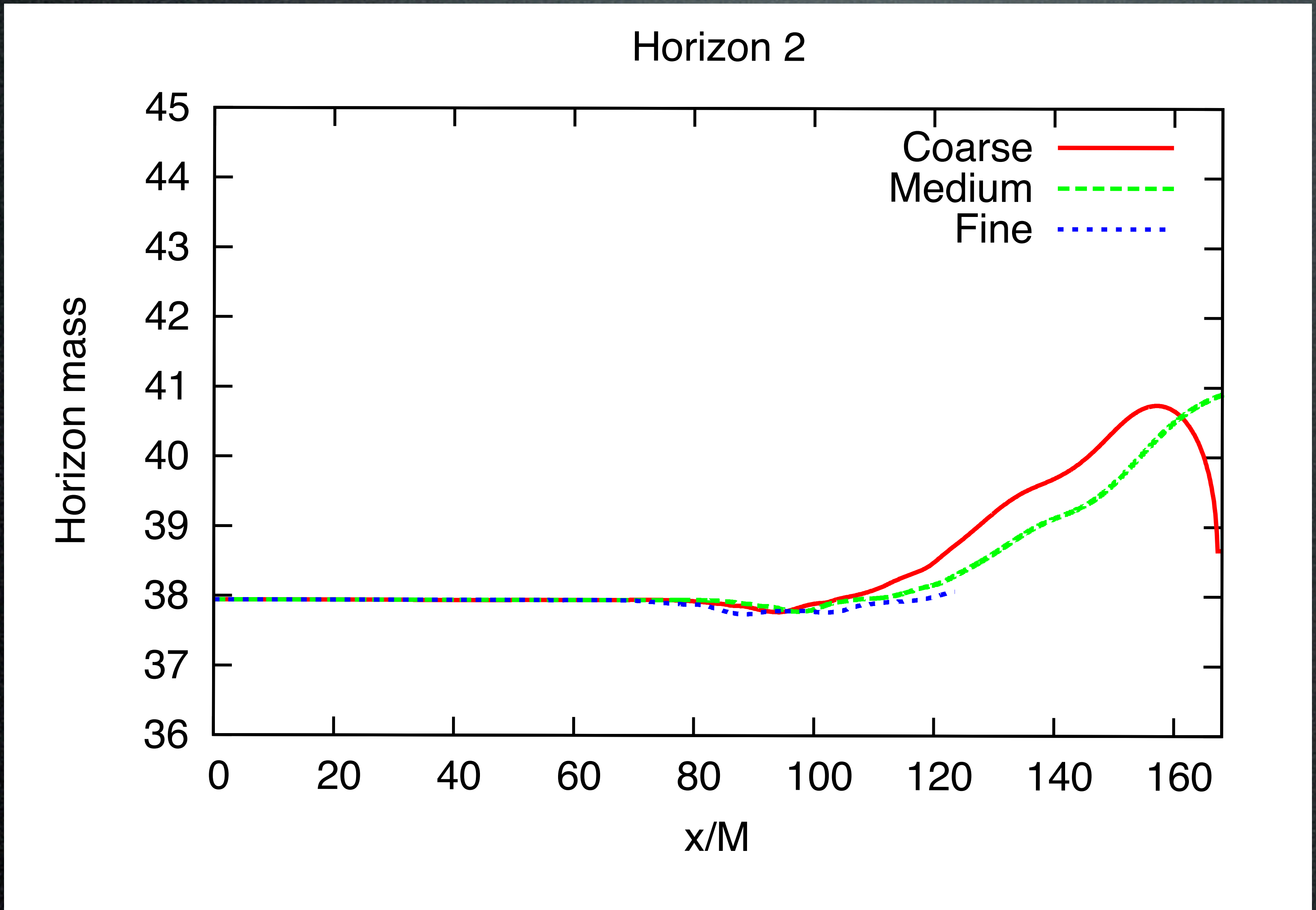


- The horizon mass is essentially constant throughout the evolution;
- Coordinate effects render the horizons harder and harder to resolve as the run progresses;
- The central horizon exhibit a jump;

BLACK-HOLE LATTICES

THE 8-BLACK-HOLE UNIVERSE

Horizon properties:



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BLACK-HOLE LATTICES

THE 8-BLACK-HOLE UNIVERSE

Harder and harder to resolve.
1) Outward shift: lack of resolution

2) Horizon instability;

BLACK-HOLE LATTICES

THE 8-BLACK-HOLE UNIVERSE

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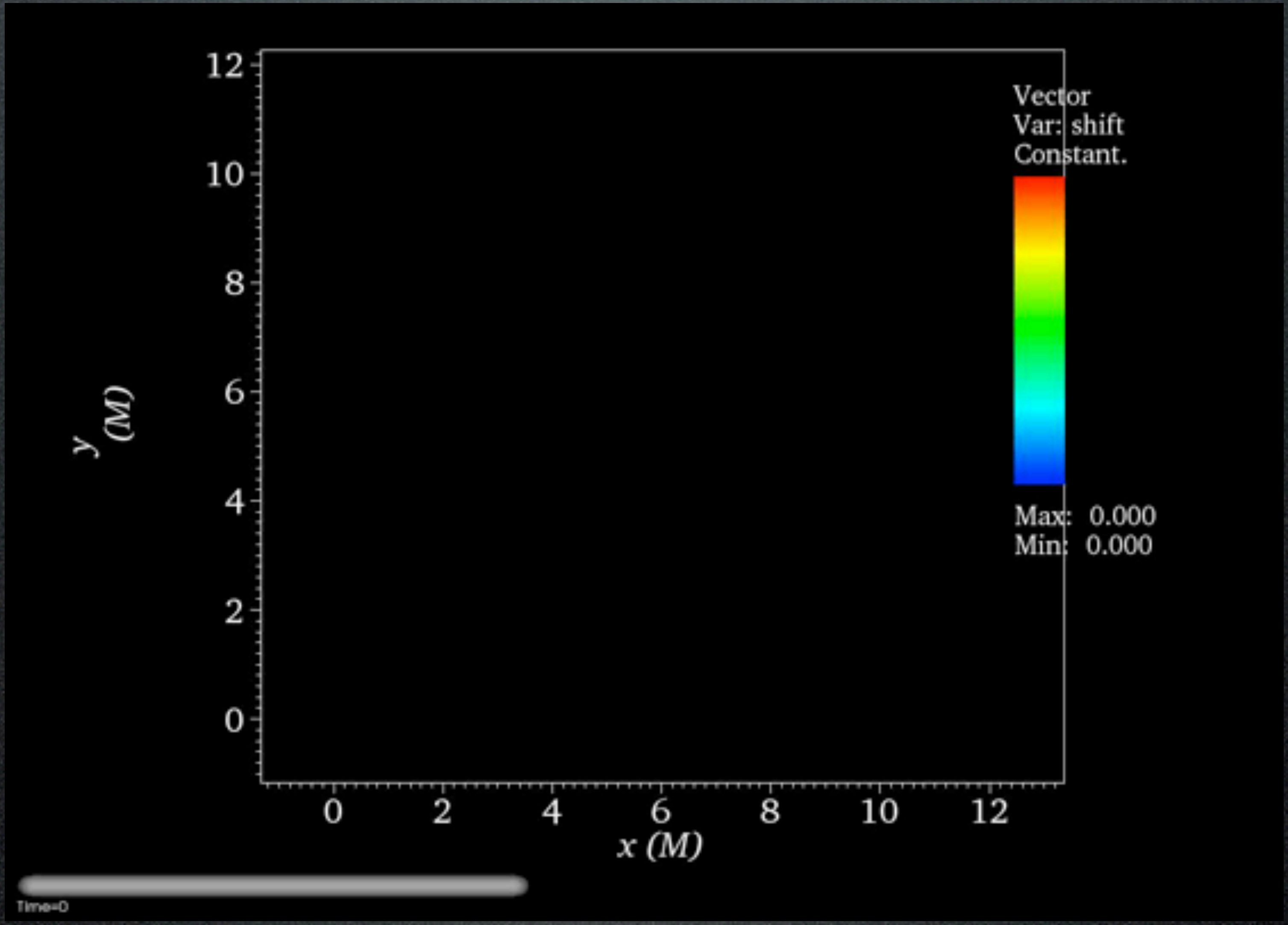

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BLACK-HOLE LATTICES

THE 8-BLACK-HOLE UNIVERSE

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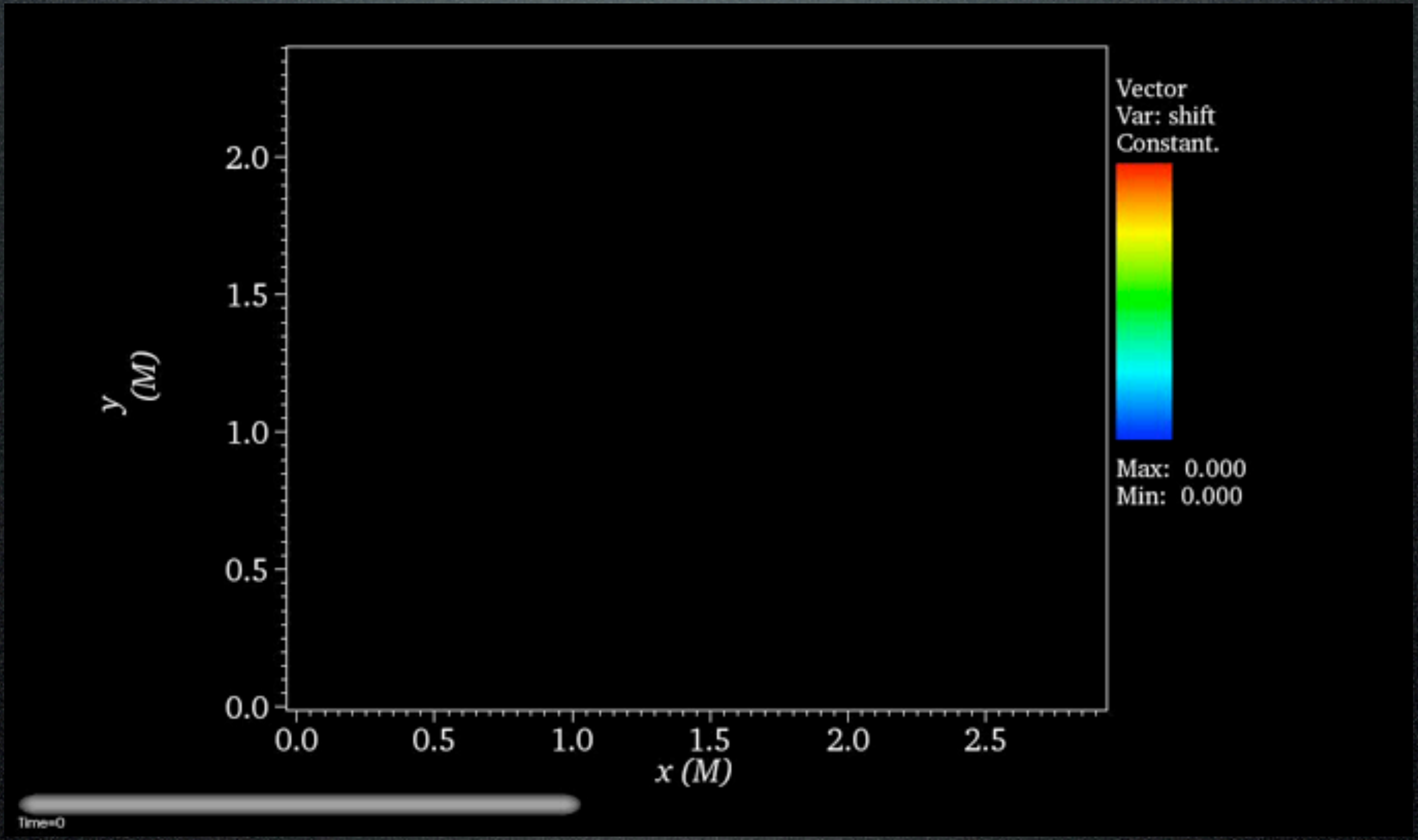

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BLACK-HOLE LATTICES

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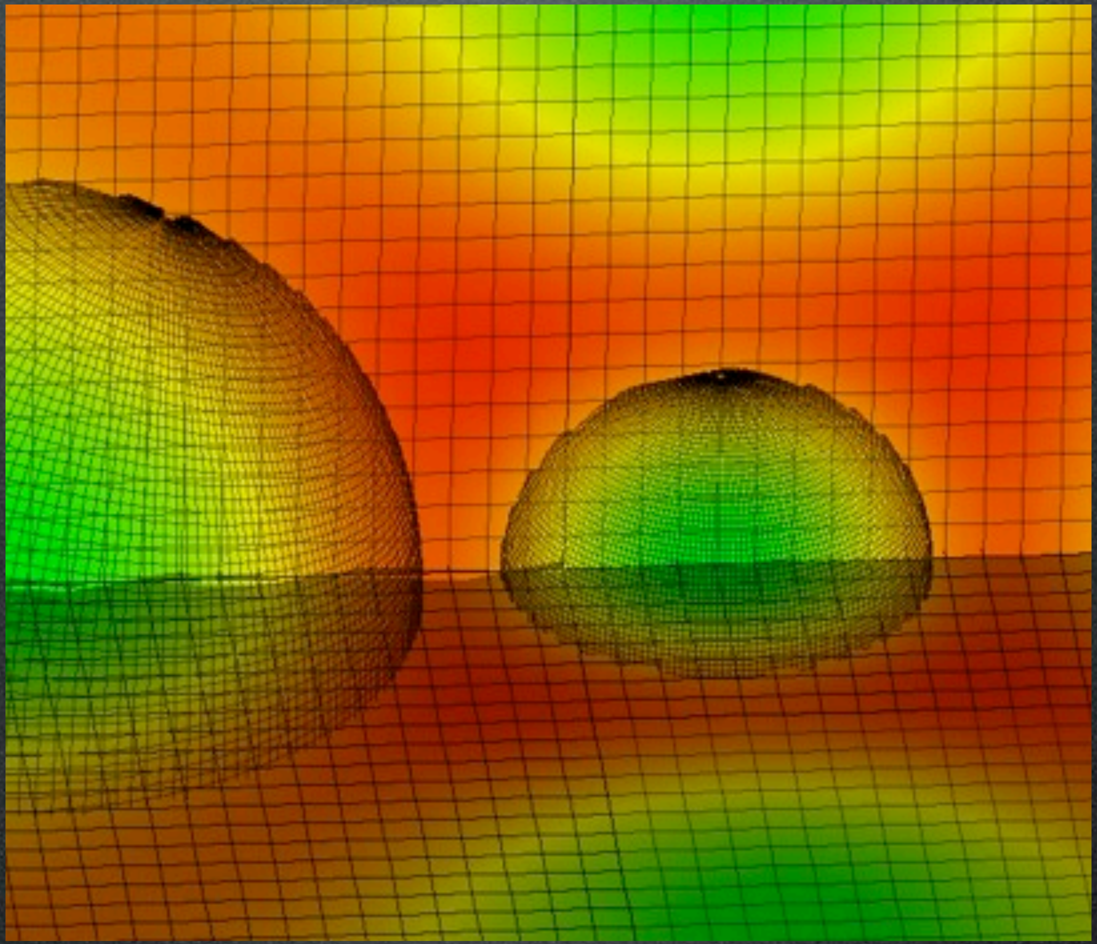
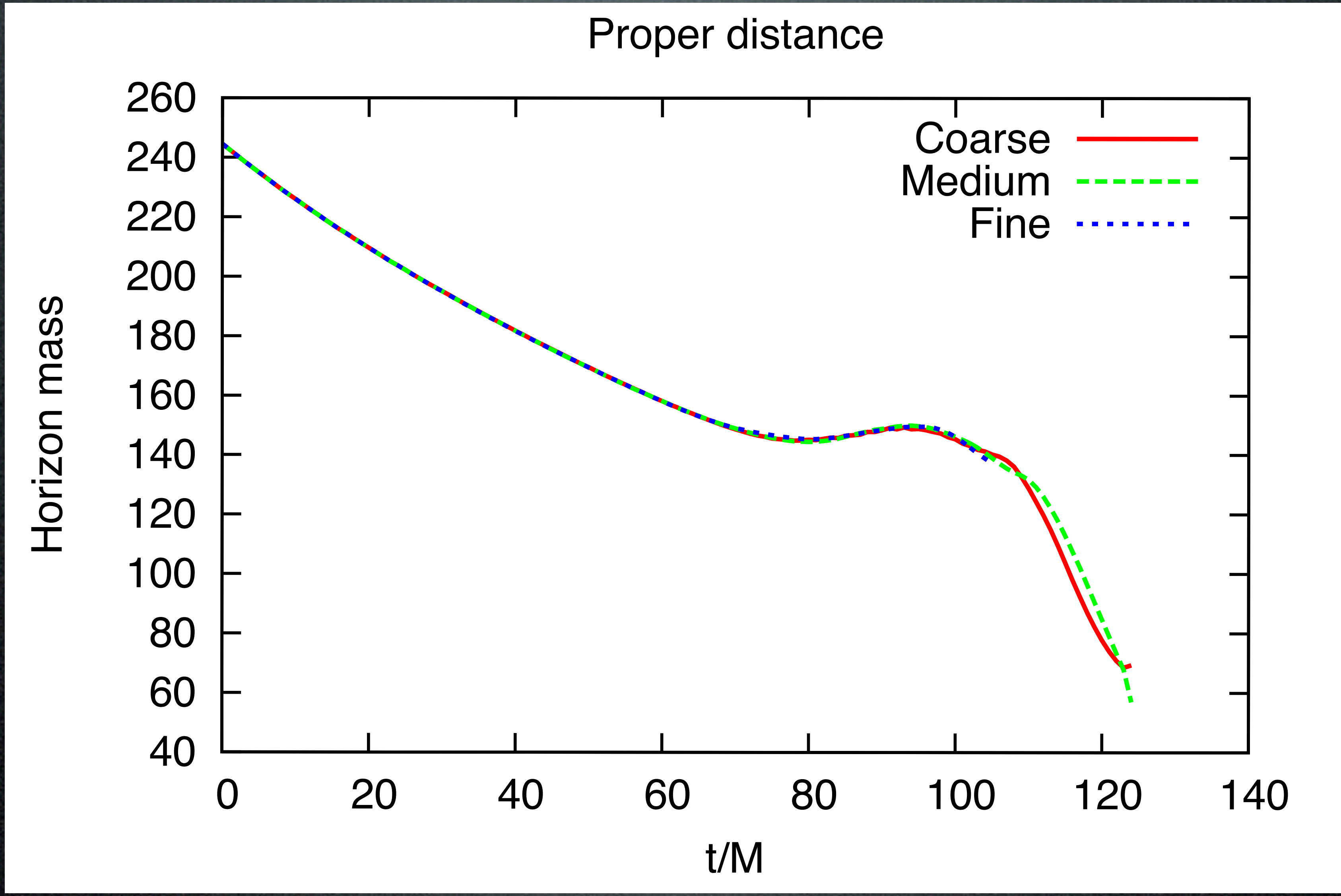


2) Horizon instability;

BLACK-HOLE LATTICES

THE 8-BLACK-HOLE UNIVERSE

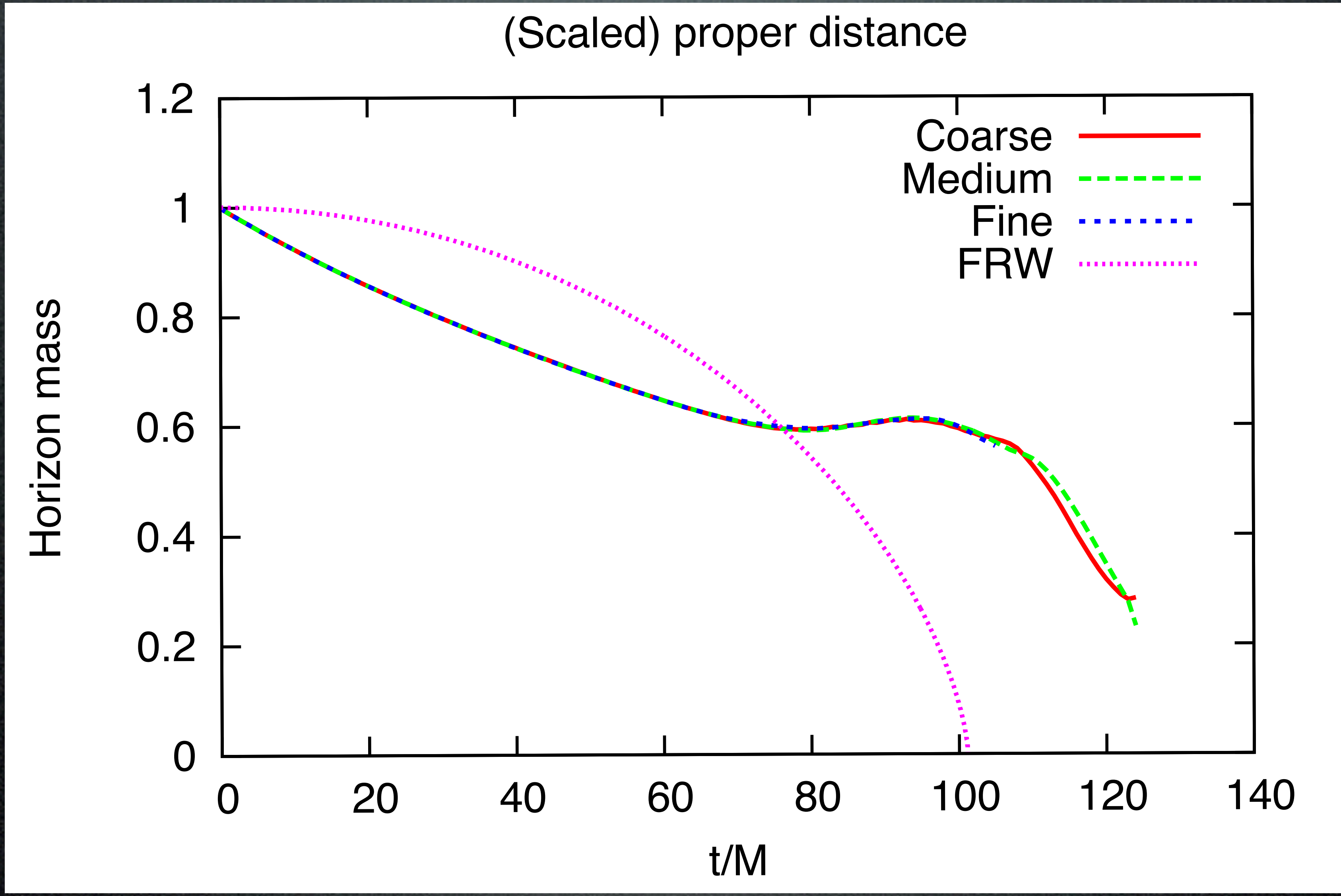
Proper distance between horizons



BLACK-HOLE LATTICES

THE 8-BLACK-HOLE UNIVERSE

Proper distance between horizons



PART IV

INHOMOGENEOUS DUST

WITH MARCO BRUNI

Inhomogeneous dust

The hydrodynamical system

From the conservation of the stress-energy tensor and the continuity equation, along with an equation of state, one can derive the PDE systems governing the evolution of rest-mass density and four-velocity.

$$\nabla_\mu T^{\mu\nu} = 0,$$
$$\nabla_\mu J^\mu = 0.$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^0}(D\sqrt{-g}) + \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}(DV^i\sqrt{-g}) = 0,$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^0}(S_\mu\sqrt{-g}) + \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}(S_\mu V^i\sqrt{-g}) + \frac{\partial p}{\partial x^\mu} + \frac{1}{2}\frac{\partial g^{\alpha\beta}}{\partial x^\mu}\frac{S_\alpha S_\beta}{S^0} = 0,$$

$$\frac{\partial}{\partial x^0}(E\sqrt{-g}) + \frac{\partial}{\partial x^i}(EV^i\sqrt{-g}) + p\frac{\partial}{\partial x^\mu}(u^0V^\mu\sqrt{-g}) = 0,$$

$$D = \rho u^0, \quad S_\mu = \rho h u_\mu u^0, \quad E = \rho\varepsilon u^0, \quad V^\mu = u^\mu/u^0$$

Inhomogeneous dust

The constraints

As usual, initial data needs to solve the constraints:

$$\tilde{\Delta}\psi - \tilde{R}\psi - \frac{K^2}{12}\psi^5 + \frac{1}{8}\tilde{A}_{ij}\tilde{A}^{ij}\psi^{-7} = -2\pi n^a n^b T_{ab}$$
$$\tilde{D}_i\tilde{A}^{ij} - \frac{2}{3}\psi^6\tilde{\gamma}^{ij}\tilde{D}_i K = -8\pi n^a T_a{}^j$$

Assume a pressureless perfect fluid:

$$T_{ab} = \rho u_a u_b$$

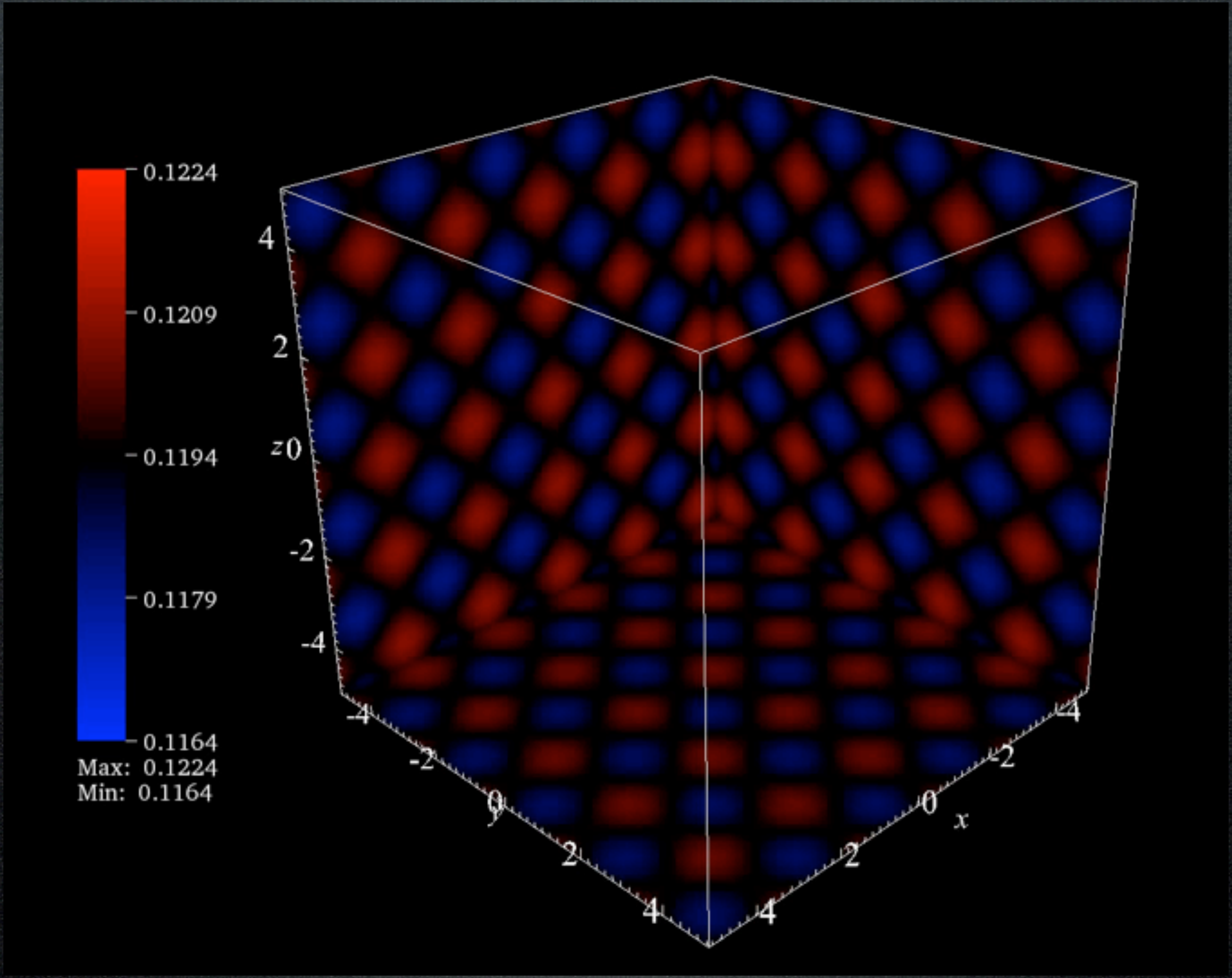
then set γ_{ij} and K_{ij} to the Einstein-deSitter solution and ρ to a perturbation of this solution:

$$\begin{aligned}
\gamma_{ij}(t_0) &= \delta_{ij} \\
K_{ij}(t_0) &= -H_{\rm EdS}(t_0)\gamma_{ij} \\
\rho(t_0, x^i) &= \rho_{\rm EdS}(t_0) + \delta_i \sin(2\pi k_{x^i} x^i)
\end{aligned}$$

Two runs: $\delta_i = 0.001$ and $\delta_i = 0.01$ (in units where $\rho_{\rm EdS}(t_0) = 0.119366$).

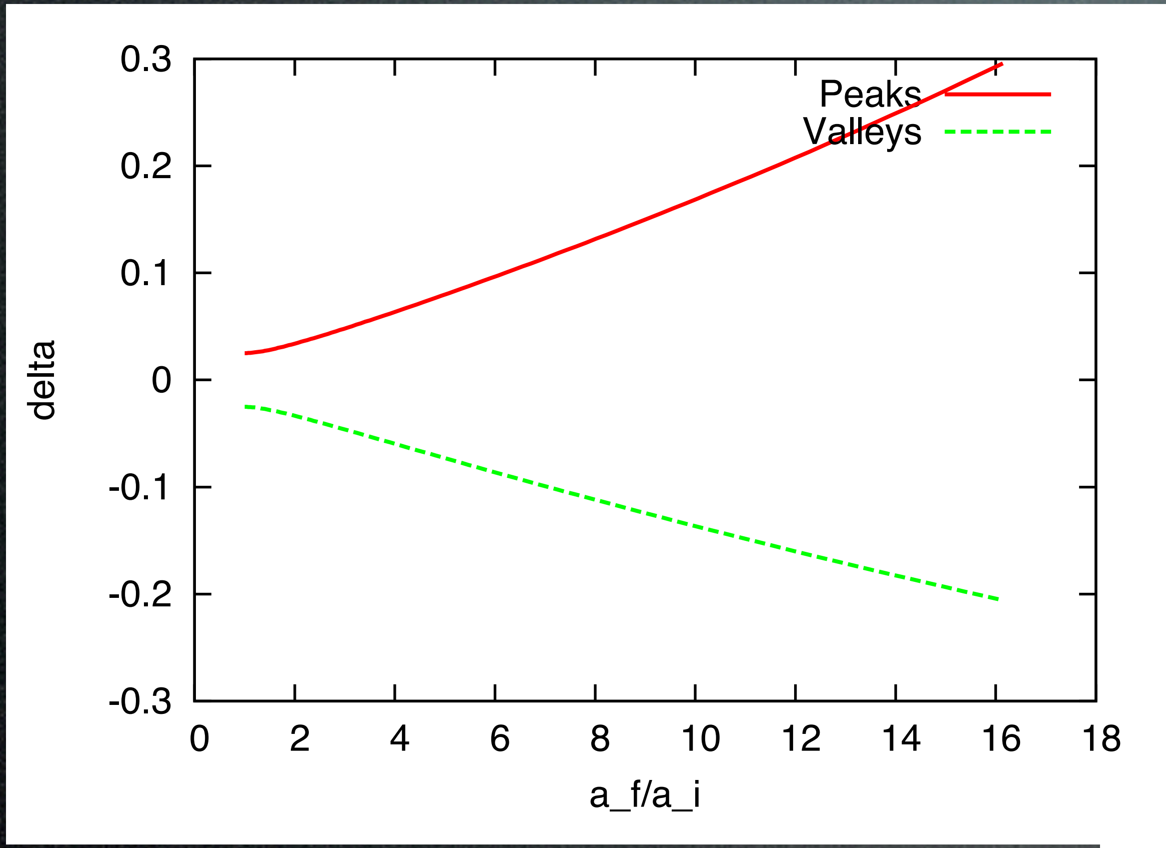
INHOMOGENEOUS DUST

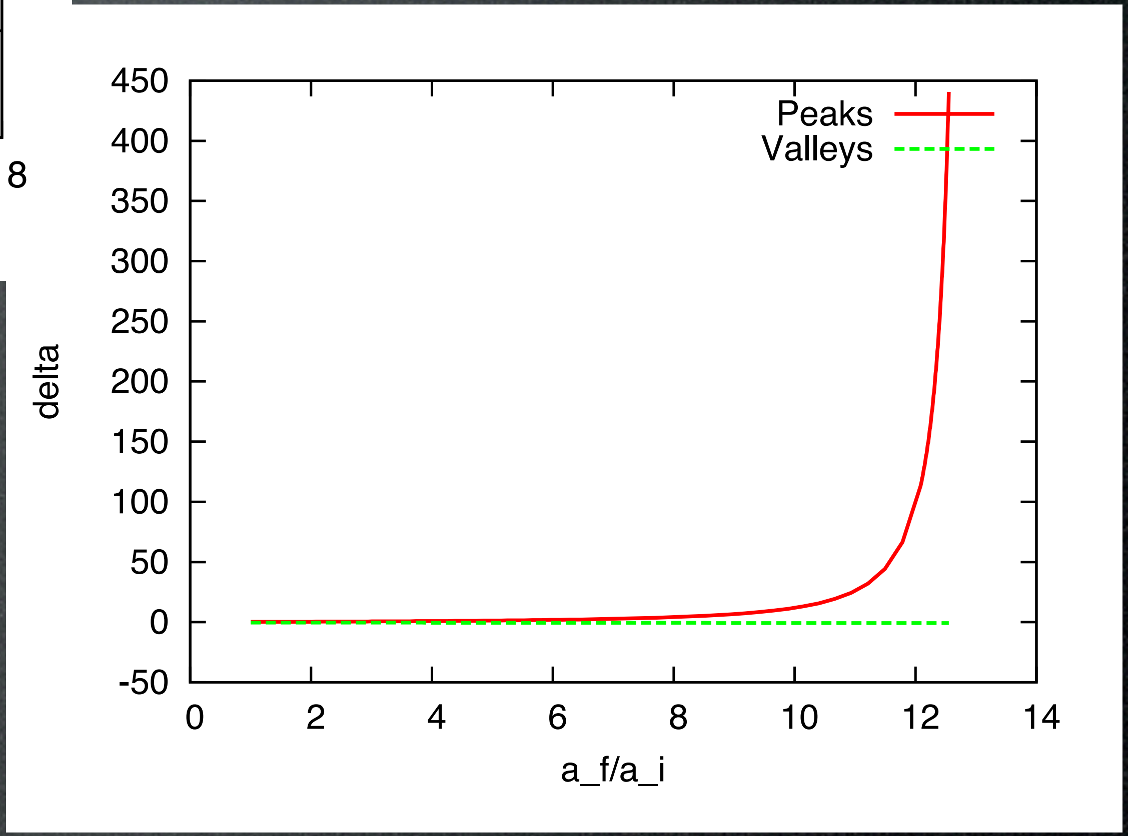
EVOLUTION OF THE DENSITY



INHOMOGENEOUS DUST

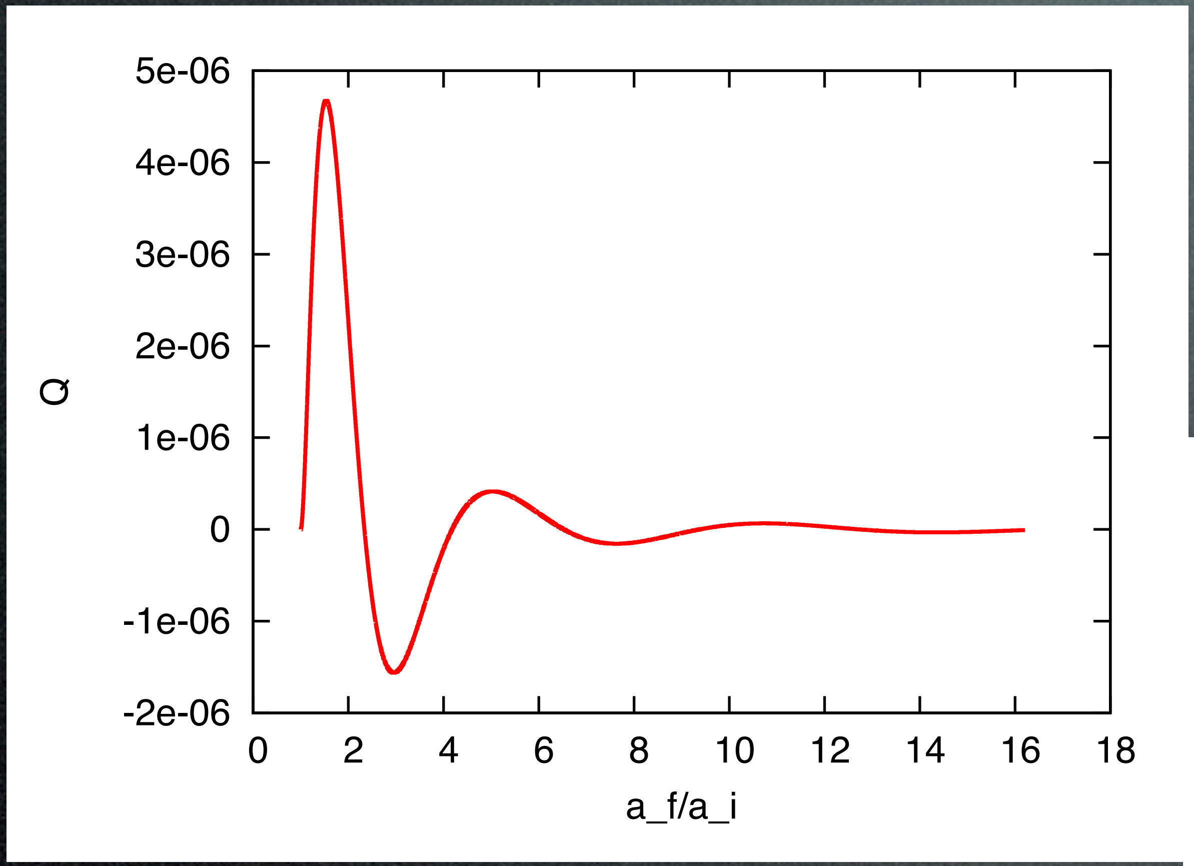
EVOLUTION OF THE DENSITY CONSTRAST

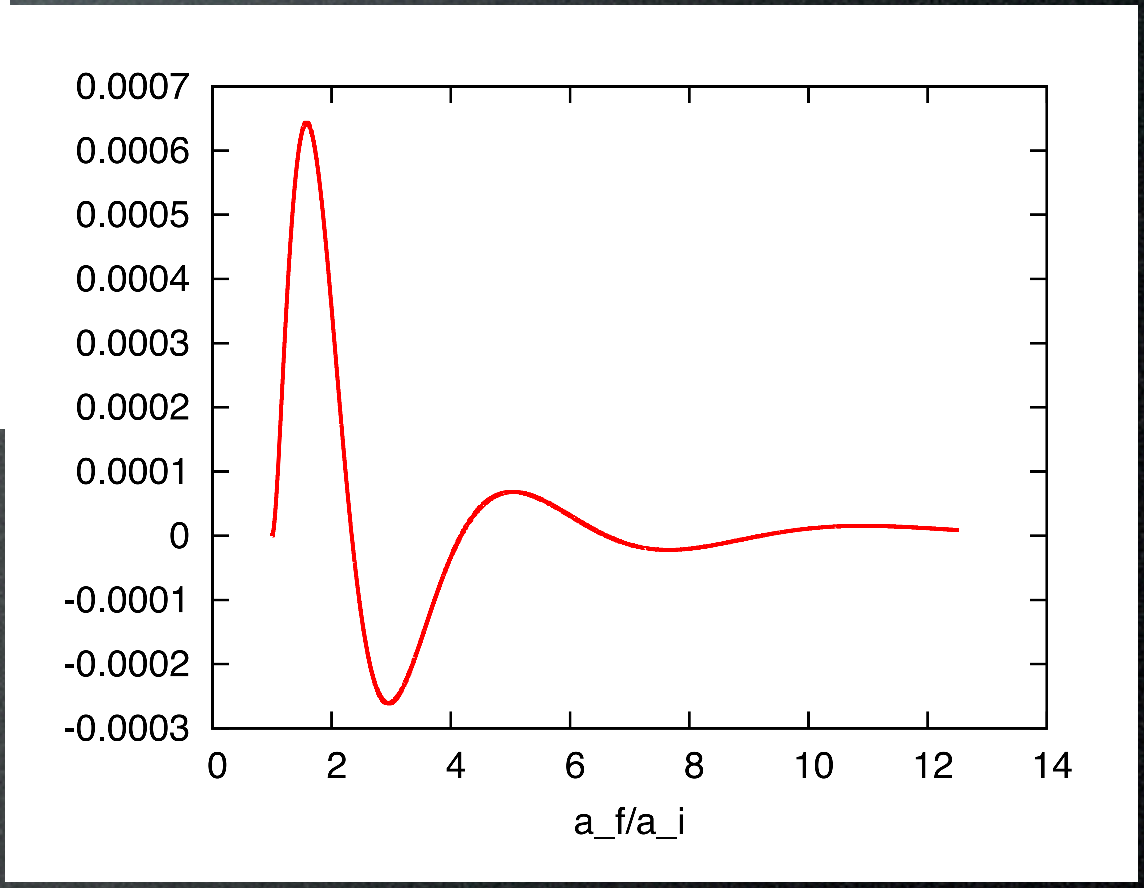




INHOMOGENEOUS DUST

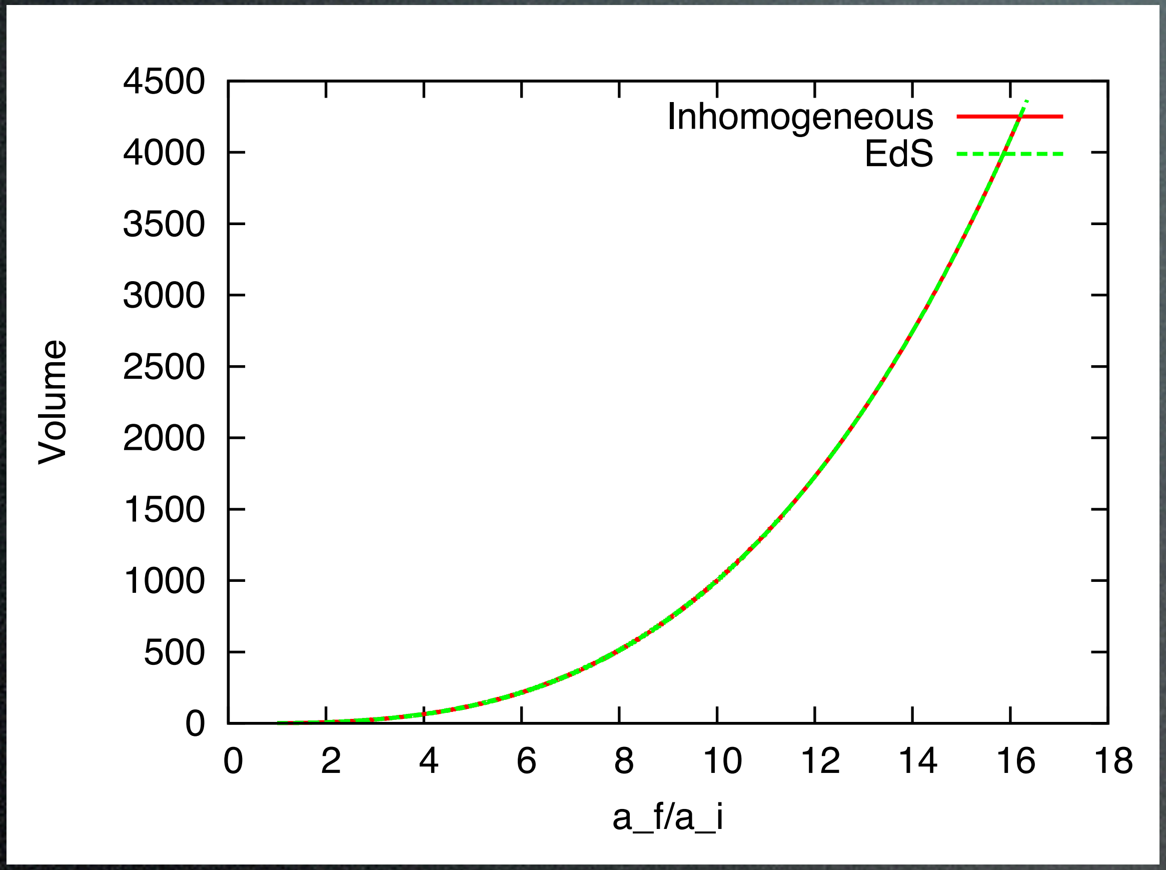
EVOLUTION OF THE BACKREACTION TERM

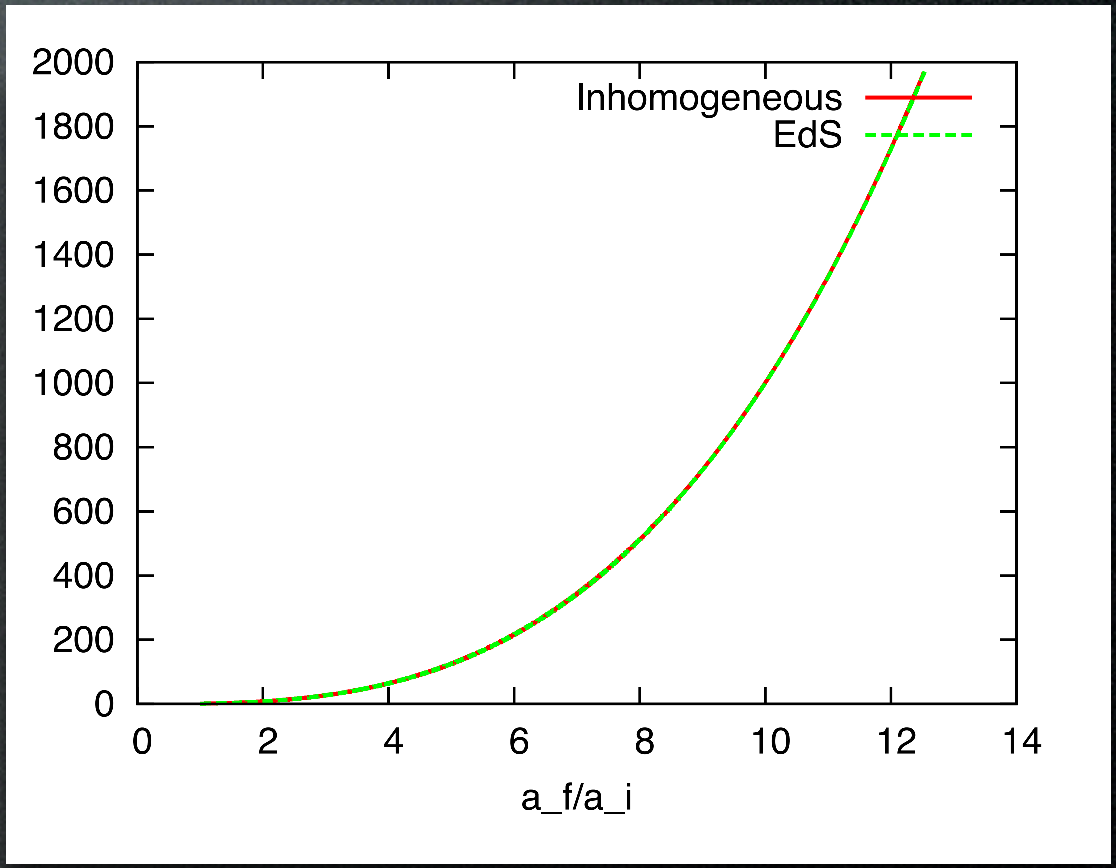




Inhomogeneous dust

Evolution of the cell volume





Conclusions and prospects

Despite the large amount of proposals, type-Ia-supernova data remain unexplained.

It is time to quantify the effect of averaging on the cosmological parameters: numerical relativity can bridge the gap between exact models and averaging schemes.

We have barely started to scratch the surface:

- Better initial data:
  - Generic solver;
  - Beyond the conformal scheme?
- Better control on the gauge used for the evolution:
  - Dust approximation leads to singularities, can these be avoided?
- More realism:
  - S^3 universes have an upper limit on the number of black holes — open three-spaces will be more realistic;
  - Scale span is tiny!